

# **UFC Specification and User Manual 1.0**

---

June 17, 2007

**Martin Sandve Alnæs, Hans Petter Langtangen,  
Anders Logg, Kent-Andre Mardal and Ola Skavhaug**

---

[www.fenics.org](http://www.fenics.org)

Visit <http://www.fenics.org/> for the latest version of this manual.  
Send comments and suggestions to [ufc-dev@fenics.org](mailto:ufc-dev@fenics.org).

# Contents

<b>1</b>	<b>Introduction</b>	<b>9</b>
1.1	Unified Form-assembly Code . . . . .	10
1.2	Aim and scope . . . . .	10
1.3	Outline . . . . .	11
<b>2</b>	<b>Finite element assembly</b>	<b>13</b>
2.1	An introductory example . . . . .	13
2.2	Discretization of variational forms . . . . .	14
2.2.1	The Finite Element . . . . .	14
2.2.2	Variational forms . . . . .	15
2.3	Assembling the discrete operator . . . . .	18
<b>3</b>	<b>Interface specification</b>	<b>23</b>
3.1	A short remark on design . . . . .	23
3.2	Cell shapes . . . . .	24

3.3	The class <code>ufc::mesh</code> . . . . .	25
3.3.1	The integer <code>topological_dimension</code> . . . . .	25
3.3.2	The integer <code>geometric_dimension</code> . . . . .	25
3.3.3	The array <code>num_entities</code> . . . . .	25
3.4	The class <code>ufc::cell</code> . . . . .	26
3.4.1	The enum variable <code>cell_shape</code> . . . . .	26
3.4.2	The integer <code>topological_dimension</code> . . . . .	26
3.4.3	The integer <code>geometric_dimension</code> . . . . .	27
3.4.4	The array <code>entity_indices</code> . . . . .	27
3.4.5	The array <code>coordinates</code> . . . . .	27
3.5	The class <code>ufc::function</code> . . . . .	28
3.5.1	The function <code>evaluate</code> . . . . .	28
3.6	The class <code>ufc::finite_element</code> . . . . .	29
3.6.1	The function <code>signature</code> . . . . .	29
3.6.2	The function <code>cell_shape</code> . . . . .	29
3.6.3	The function <code>space_dimension</code> . . . . .	29
3.6.4	The function <code>value_rank</code> . . . . .	30
3.6.5	The function <code>value_dimension</code> . . . . .	30
3.6.6	The function <code>evaluate_basis</code> . . . . .	30
3.6.7	The function <code>evaluate_basis_derivatives</code> . . . . .	31
3.6.8	The function <code>evaluate_dof</code> . . . . .	32

3.6.9	The function <code>interpolate_vertex_values</code> . . . . .	32
3.6.10	The function <code>num_sub_elements</code> . . . . .	32
3.6.11	The function <code>create_sub_element</code> . . . . .	33
3.7	The class <code>ufc::dof_map</code> . . . . .	33
3.7.1	The function <code>signature</code> . . . . .	34
3.7.2	The function <code>needs_mesh_entities</code> . . . . .	34
3.7.3	The function <code>init_mesh</code> . . . . .	34
3.7.4	The function <code>init_cell</code> . . . . .	35
3.7.5	The function <code>init_cell_finalize</code> . . . . .	35
3.7.6	The function <code>global_dimension</code> . . . . .	35
3.7.7	The function <code>local_dimension</code> . . . . .	36
3.7.8	The function <code>num_facet_dofs</code> . . . . .	36
3.7.9	The function <code>tabulate_dofs</code> . . . . .	36
3.7.10	The function <code>tabulate_facet_dofs</code> . . . . .	36
3.7.11	The function <code>tabulate_coordinates</code> . . . . .	37
3.7.12	The function <code>num_sub_dof_maps</code> . . . . .	37
3.7.13	The function <code>create_sub_dof_map</code> . . . . .	38
3.8	The integral classes . . . . .	38
3.9	The class <code>ufc::cell_integral</code> . . . . .	39
3.9.1	The function <code>tabulate_tensor</code> . . . . .	39
3.10	The class <code>ufc::exterior_facet_integral</code> . . . . .	39

3.10.1	The function <code>tabulate_tensor</code> . . . . .	40
3.11	The class <code>ufc::interior_facet_integral</code> . . . . .	40
3.11.1	The function <code>tabulate_tensor</code> . . . . .	40
3.12	The class <code>ufc::form</code> . . . . .	42
3.12.1	The function <code>signature</code> . . . . .	42
3.12.2	The function <code>rank</code> . . . . .	42
3.12.3	The function <code>num_coefficients</code> . . . . .	43
3.12.4	The function <code>num_cell_integrals</code> . . . . .	43
3.12.5	The function <code>num_exterior_facet_integrals</code> . . . . .	43
3.12.6	The function <code>num_interior_facet_integrals</code> . . . . .	43
3.12.7	The function <code>create_finite_element</code> . . . . .	44
3.12.8	The function <code>create_dof_map</code> . . . . .	44
3.12.9	The function <code>create_cell_integral</code> . . . . .	44
3.12.10	The function <code>create_exterior_facet_integral</code> . . . . .	45
3.12.11	The function <code>create_interior_facet_integral</code> . . . . .	45
<b>4</b>	<b>Reference cells</b> . . . . .	<b>47</b>
4.1	The reference interval . . . . .	48
4.2	The reference triangle . . . . .	49
4.3	The reference quadrilateral . . . . .	50
4.4	The reference tetrahedron . . . . .	51

4.5	The reference hexahedron . . . . .	52
<b>5</b>	<b>Numbering of mesh entities</b>	<b>53</b>
5.1	Basic concepts . . . . .	53
5.2	Numbering of vertices . . . . .	54
5.3	Numbering of other mesh entities . . . . .	54
5.3.1	Relative ordering . . . . .	56
5.3.2	Limitations . . . . .	57
5.4	Numbering schemes for reference cells . . . . .	58
5.4.1	Numbering for intervals . . . . .	58
5.4.2	Numbering for triangular cells . . . . .	59
5.4.3	Numbering for quadrilateral cells . . . . .	59
5.4.4	Numbering for tetrahedral cells . . . . .	60
5.4.5	Numbering for hexahedral cells . . . . .	61
<b>A</b>	<b>C++ Interface</b>	<b>65</b>
<b>B</b>	<b>A basic UFC-based assembler</b>	<b>73</b>
<b>C</b>	<b>Complete UFC code for Poisson's equation</b>	<b>77</b>
C.1	Code generated by FFC . . . . .	78
C.2	Code generated by SyFi . . . . .	91
C.2.1	Header file for linear Lagrange element in 2D . . . . .	91

C.2.2	Source file for linear Lagrange element in 2D . . . . .	93
C.2.3	Header file for the dofmap . . . . .	96
C.2.4	Source file for the dofmap . . . . .	97
C.2.5	Header file for the stiffness matrix form . . . . .	101
C.2.6	Source file for the stiffness matrix form . . . . .	103
<b>D</b>	<b>Python utilities</b>	<b>107</b>
<b>E</b>	<b>Installation</b>	<b>109</b>
E.1	Dependencies and requirements . . . . .	109
E.1.1	Installing Python . . . . .	110
E.2	Installing UFC . . . . .	110
<b>F</b>	<b>License</b>	<b>111</b>



# Chapter 1

## Introduction

Large parts of a finite element program are similar from problem to problem and can therefore be coded as a general, reusable library. Mesh data structures, linear algebra and finite element assembly are examples of operations that are naturally coded in a problem-independent way and made available in reusable libraries [7, 3, 16, 4, 1, 5]. However, some parts of a finite element program are difficult to code in a problem-independent way. In particular, this includes the evaluation of the *element tensor* (the “element stiffness matrix”), that is, the evaluation of the local contribution from a finite element to a global sparse tensor (the “stiffness matrix”) representing a discretized differential operator. These parts must thus be implemented by the application programmer for each specific combination of differential equation and discretization (finite element spaces).

However, domain-specific compilers such as FFC [15, 10, 13, 11, 12] and SyFi [2] make it possible to automatically generate the code for the evaluation of the element tensor. These *form compilers* accept as input a high-level description of a finite element variational form and generate low-level code for efficient evaluation of the element tensor and associated quantities. It thus becomes important to specify the *interface* between form compilers and finite element assemblers such that the code generated by FFC, SyFi and other form compilers can be used to assemble finite element matrices and vectors (and in general tensors).

### 1.1 Unified Form-assembly Code

UFC (Unified Form-assembly Code) is a unified framework for finite element assembly. More precisely, it defines a fixed interface for communicating low level routines (functions) for evaluating and assembling finite element variational forms. The UFC interface consists of a single header file `ufc.h` that specifies a C++ interface that must be implemented by code that complies with the UFC specification.

Both FFC (since version 0.4.0) and SyFi (since version 0.3.4) generate code that complies with the UFC specification. Thus, code generated by FFC and SyFi may be used interchangeably by any UFC-based finite element assembler, such as DOLFIN [8].

### 1.2 Aim and scope

The UFC interface has been designed to make a minimal amount of assumptions on the form compilers generating the UFC code and the assemblers built on top of the UFC specification. Thus, the UFC specification provides a minimal amount of abstractions and data structures. Programmers wishing to implement the UFC specification will typically want to create system-specific (but simple) wrappers for the generated code.

Few assumptions have also been made on the underlying finite element methodology. The current specification is limited to affinely mapped cells, but does not restrict the mapping of finite element function spaces. Thus, UFC code may be generated for elements where basis functions are transformed from the reference cell by the affine mapping, as well as for elements where the basis functions must be transformed by the Piola mapping. UFC code has been successfully generated and used in finite element codes for standard continuous Galerkin methods (Lagrange finite elements of arbitrary order), discontinuous Galerkin methods (including integrals of jumps and averages over interior facets) and mixed methods (including Brezzi–Douglas–Marini and Raviart–Thomas elements).

## **1.3 Outline**

In the next section, we give an overview of finite element assembly and explain how the code generated by form compilers may be used as the basic building blocks in the assembly algorithm. We then present the UFC interface in detail in Section 3. In Section 4 and Section 5, we define the reference cells and numbering conventions that must be followed by UFC-based form compilers and assemblers.



# Chapter 2

## Finite element assembly

In this section, we present a general algorithm for assembly of finite element variational forms and define the concepts that the UFC interface is based on.

### 2.1 An introductory example

As an introduction, consider Poisson's equation,

$$\begin{aligned} -\Delta u &= f, & \text{in } \Omega, \\ u &= 0, & \text{on } \partial\Omega. \end{aligned} \tag{2.1}$$

For  $v \in \hat{V}$  a test function, we multiply (2.1) by  $v$  and integrate by parts to obtain the variational problem: Find  $u \in V$  such that

$$\int_{\Omega} \nabla v \cdot \nabla u \, dx = \int_{\Omega} v f \, dx \quad \forall v \in \hat{V}, \tag{2.2}$$

which we may restate as: Find  $u \in V$  such that

$$a(v, u) = L(v) \quad \forall v \in \hat{V}, \tag{2.3}$$

where the *bilinear form*  $a : \hat{V} \times V \rightarrow \mathbb{R}$  is given by

$$a(v, u) = \int_{\Omega} \nabla v \cdot \nabla u \, dx, \tag{2.4}$$

## UFC Specification and User Manual 1.0

---

and the *linear form*  $L : \hat{V} \rightarrow \mathbb{R}$  is given by

$$L(v) = \int_{\Omega} v f \, dx. \quad (2.5)$$

By replacing the test space  $\hat{V}$  and the trial space  $V$  with a pair of discrete test and trial spaces  $\hat{V}_h \subset \hat{V}$  and  $V_h \subset V$ , we obtain the discrete variational problem: Find  $u_h \in V_h$  such that

$$a(v, u_h) = L(v) \quad \forall v \in \hat{V}_h. \quad (2.6)$$

To obtain the discrete solution  $u_h \in V_h$ , we make an ansatz for  $u_h$  to be expressed as a linear combination of a suitable set of basis functions  $\{\phi_j\}_{j=1}^N$  for  $V_h$ ,

$$u_h(x) = \sum_{j=1}^N U_j \phi_j(x), \quad (2.7)$$

and take  $v = \hat{\phi}_i$  for  $i = 1, 2, \dots, N$  with  $\{\hat{\phi}_i\}_{i=1}^N$  a set of basis functions for the test space  $\hat{V}_h$ . We thus obtain the linear system

$$a(\hat{\phi}_i, \sum_{j=1}^N U_j \phi_j) = L(\hat{\phi}_i), \quad i = 1, 2, \dots, N, \quad (2.8)$$

or, since  $a$  is bilinear,

$$AU = b, \quad (2.9)$$

where

$$A_{ij} = a(\hat{\phi}_i, \phi_j), \quad i = 1, 2, \dots, N, j = 1, 2, \dots, N, \quad (2.10)$$

$$b_i = L(\hat{\phi}_i), \quad i = 1, 2, \dots, N. \quad (2.11)$$

## 2.2 Discretization of variational forms

### 2.2.1 The Finite Element

A finite element is mathematically defined as a triplet consisting of a polygon, a function space, and a set of linear functionals. Given that the dimension of

---

## UFC Specification and User Manual 1.0

---

the function space and the number of the (linearly independent) linear functionals are equal, the finite element is uniquely defined. With this definition of a finite element, the basis functions are implicitly defined as the solution of a linear system.

We take our starting point in the standard Ciarlet definition [6], but consider a finite element with a fixed set of basis functions. These basis functions are pre-computed from the triplet introduced above. Hence, we will refer to a finite element as a collection of

- a polygon  $K$ ,
- a set of basis functions  $\{\phi_i\}_{i=1}^n$  spanning  $\mathcal{P}_K$  and the derivatives of those basis functions,
- the degrees of freedom, which are linear functionals,  $L_i : V \rightarrow \mathbb{R}, i = 1, 2, \dots, n$ , where  $V = \text{span}\{\phi_i\}_{i=1}^n$ . Usually,  $L_i$  may be extended from  $V$  to an appropriate Sobolev space.

In addition, our finite element will contain functionality for interpolation.

### 2.2.2 Variational forms

In general, we shall be concerned with the discretization of general variational forms of arity  $r > 0$ ,

$$a : V_h^1 \times V_h^2 \times \dots \times V_h^r \times W_h^1 \times W_h^2 \times \dots \times W_h^n \rightarrow \mathbb{R}. \quad (2.12)$$

defined on the product space  $V_h^1 \times V_h^2 \times \dots \times V_h^r \times W_h^1 \times W_h^2 \times \dots \times W_h^n$  of two sets  $\{V_h^j\}_{j=1}^r, \{W_h^j\}_{j=1}^n$  of discrete function spaces on a triangulation  $\mathcal{T}$  of a domain  $\Omega \subset \mathbb{R}^d$ . We also often write this as

$$a = a(v_1, \dots, v_r; w_1, \dots, w_n), \quad (2.13)$$

where ‘;’ is used to separate the  $V_h$  spaces from the  $W_h$  spaces. The distinction between the spaces  $\{V_h^j\}_{j=1}^r$  and  $\{W_h^j\}_{j=1}^n$  lies in how each set of spaces is discretized. This will be explained in the following.

---

## UFC Specification and User Manual 1.0

---

In the simplest case, all function spaces are equal but there are many important examples, such as mixed methods, where it is important to consider arguments coming from different function spaces.

Let  $\mathcal{T} = \{K\}$  be a set of disjoint *cells* (a triangulation) partitioning a domain  $\cup_{K \in \mathcal{T}} K = \Omega \subset \mathbb{R}^d$ . Further, let  $\partial_e \mathcal{T}$  denote the set of *exterior facets* (the set of cell facets incident with the boundary  $\partial\Omega$ ), and let  $\partial_i \mathcal{T}$  denote the set of *interior facets* (the set of cell facets non-incident with the boundary  $\partial\Omega$ ).

We shall assume that the variational form (2.12) may be expressed as a sum of integrals over the cells  $\mathcal{T}$ , the exterior facets  $\partial_e \mathcal{T}$  and the interior facets  $\partial_i \mathcal{T}$ . We shall allow integrals expressed on disjoint subsets  $\{\mathcal{T}_k\}_{k=1}^{n_c}$ ,  $\{\partial_e \mathcal{T}_k\}_{k=1}^{n_e}$  and  $\{\partial_i \mathcal{T}_k\}_{k=1}^{n_i}$  of  $\mathcal{T}$ ,  $\partial_e \mathcal{T}$  and  $\partial_i \mathcal{T}$  respectively.

We thus assume that the form  $a$  is given by

$$\begin{aligned} a(v_1, \dots, v_r; w_1, \dots, w_n) &= \sum_{k=1}^{n_c} \sum_{K \in \mathcal{T}_k} \int_K I_k^c(v_1, \dots, v_r; w_1, \dots, w_n) \, dx \\ &\quad + \sum_{k=1}^{n_e} \sum_{S \in \partial_e \mathcal{T}_k} \int_S I_k^e(v_1, \dots, v_r; w_1, \dots, w_n) \, ds \quad (2.14) \\ &\quad + \sum_{k=1}^{n_i} \sum_{S \in \partial_i \mathcal{T}_k} \int_S I_k^i(v_1, \dots, v_r; w_1, \dots, w_n) \, ds. \end{aligned}$$

We refer to an integral over a cell  $K$  as a *cell integral*, an integral over an exterior facet  $S$  as an *exterior facet integral*, and to an integral over an interior facet  $S$  as an *interior facet integral*.

To discretize the form  $a$ , we let  $\{\phi_i^1\}_{i=1}^{N^1}, \{\phi_i^2\}_{i=1}^{N^2}, \dots, \{\phi_i^r\}_{i=1}^{N^r}$  be bases of  $V_h^1, V_h^2, \dots, V_h^r$  respectively and let  $i = (i_1, i_2, \dots, i_r)$  be a multiindex of length  $|i| = r$ . The form  $a$  then defines a rank  $r$  tensor given by

$$A_i = a(\phi_{i_1}^1, \phi_{i_2}^2, \dots, \phi_{i_r}^r; w_1, w_2, \dots, w_n) \quad \forall i \in \mathcal{I}, \quad (2.15)$$

where  $\mathcal{I}$  is the index set

$$\mathcal{I} = \prod_{j=1}^r [1, |V_h^j|] = \{(1, 1, \dots, 1), (1, 1, \dots, 2), \dots, (N^1, N^2, \dots, N^r)\}. \quad (2.16)$$



## UFC Specification and User Manual 1.0

---

For any given form of arity  $r$ , the tensor  $A$  is a (typically sparse) tensor of rank  $r$  and dimension  $|V_h^1| \times |V_h^2| \times \dots \times |V_h^r| = N^1 \times N^2 \times \dots \times N^r$ .

Typically, the arity of the form  $a$  is  $r = 0$ ,  $r = 1$  or  $r = 2$ . When  $r = 0$ , the corresponding (rank zero) tensor is a scalar, when  $r = 1$ , the corresponding (rank one) tensor is a vector (the “load vector”) and when  $r = 2$ , the corresponding (rank two) tensor is a matrix (the “stiffness matrix”). Forms of higher arity also appear, though they are rarely assembled as a higher-dimensional sparse tensor.

Note here that we consider the functions  $w_1, w_2, \dots, w_n$  as fixed in the sense that the discrete operator is computed for a given set of functions, which we refer to as *coefficients*. As an example, consider again the variational problem (2.2) for Poisson’s equation. For the bilinear form  $a$ , the arity (rank) is  $r = 2$  and the number of coefficients is  $n = 0$ , while for the linear form  $L$ , the arity is  $r = 1$  and the number of coefficients is  $n = 1$ . We may also choose to directly compute the *action* of the bilinear form  $a$  obtained by assembling a vector from the form

$$a(v_1; w_1) = \int_{\Omega} \nabla v_1 \cdot \nabla w_1 \, dx, \quad (2.17)$$

where now  $r = 1$  and  $n = 1$ .

We list below a few other examples to illustrate the notation.

**EXAMPLE 2.2.1** *Our first example is related to the divergence constraint in fluid flow. Let  $a$  be given by*

$$a(q, u) = \int_{\Omega} q \nabla \cdot u \, dx, \quad q \in V_h^1, \quad u \in V_h^2, \quad (2.18)$$

where  $V_h^1$  is a space of scalar-valued functions and where  $V_h^2$  is a space of vector-valued functions. The form  $a : V_h^1 \times V_h^2 \rightarrow \mathbb{R}$  is bilinear and thus  $r = 2$ . Furthermore, the form does not depend on any coefficients and thus  $n = 0$ .

**EXAMPLE 2.2.2** *Another common form in fluid flow (with variable density) is:*

$$a(v, u; w, \varrho) = \int_{\Omega} v \varrho w \cdot \nabla u \, dx. \quad (2.19)$$

## UFC Specification and User Manual 1.0

---

Here,  $v \in V_h^1$ ,  $u \in V_h^2$ ,  $w \in W_h^1$ ,  $\varrho \in W_h^2$ , where  $V_h^1$ ,  $V_h^2$ , and  $W_h^1$  are spaces of vector-valued functions, while  $W_h^2$  is a space of scalar-valued functions. The form takes four arguments, where two of the arguments are coefficients,

$$a : V_h^1 \times V_h^2 \times W_h^1 \times W_h^2 \rightarrow \mathbb{R}. \quad (2.20)$$

Hence,  $r = 2$  and  $n = 2$ .

**EXAMPLE 2.2.3** *The form corresponding to the right-hand side (of, e.g., Poisson's equation) is given by*

$$a(v; f) = \int_{\Omega} v f \, dx. \quad (2.21)$$

The form takes two arguments and one of them is a coefficient,

$$a : V_h^1 \times W_h^1 \rightarrow \mathbb{R}. \quad (2.22)$$

Hence,  $r = 1$  and  $n = 1$ .

**EXAMPLE 2.2.4** *The  $H^1(\Omega)$  norm of the error  $e = u_h - u$  squared is:*

$$a(u_h, u) = \int_{\Omega} (u_h - u)^2 + |\nabla(u_h - u)|^2 \, dx. \quad (2.23)$$

The form takes two arguments and both are coefficients,

$$a : W_h^1 \times W_h^2 \rightarrow \mathbb{R}. \quad (2.24)$$

Hence,  $r = 0$  and  $n = 2$ .

## 2.3 Assembling the discrete operator

The standard algorithm [17, 9, 14] for computing the global tensor  $A$  is known as *assembly*;  $A$  is computed by iterating over the cells  $\mathcal{T}$ , the exterior facets  $\partial_e \mathcal{T}$  and interior facets  $\partial_i \mathcal{T}$ , and adding from each the local contributions.

## UFC Specification and User Manual 1.0

---

For illustration purposes, consider the assembly of the global sparse tensor  $A$  corresponding to a form  $a$  given by a single integral over all cells  $\mathcal{T}$ ,

$$a(v_1, \dots, v_r; w_1, \dots, w_n) = \sum_{K \in \mathcal{T}} \int_K I^c(v_1, \dots, v_r; w_1, \dots, w_n) \, dx. \quad (2.25)$$

The global sparse tensor  $A$  is then given by

$$A_i = \sum_{K \in \mathcal{T}} \int_K I^c(\phi_{i_1}^1, \dots, \phi_{i_r}^r; w_1, \dots, w_n) \, dx. \quad (2.26)$$

To see how to compute the tensor  $A$  by summing the local contributions from each cell  $K$ , we let  $n_K^j = |\mathcal{P}_K^j|$  denote the dimension of the local finite element space on  $K$ . Furthermore, let

$$\iota_K^j : [1, n_K^j] \rightarrow [1, N^j] \quad (2.27)$$

denote the local-to-global mapping for each discrete function space  $V_h^j$ ,  $j = 1, 2, \dots, r$ , and define for each  $K \in \mathcal{T}$  the collective local-to-global mapping  $\iota_K : \mathcal{I}_K \rightarrow \mathcal{I}$  by

$$\iota_K(i) = (\iota_K^1(i_1), \iota_K^2(i_2), \dots, \iota_K^r(i_r)) \quad \forall i \in \mathcal{I}_K, \quad (2.28)$$

where  $\mathcal{I}_K$  is the index set

$$\mathcal{I}_K = \prod_{j=1}^r [1, |\mathcal{P}_K^j|] = \{(1, 1, \dots, 1), (1, 1, \dots, 2), \dots, (n_K^1, n_K^2, \dots, n_K^r)\}. \quad (2.29)$$

Furthermore, for each  $V_h^j$  we let  $\{\phi_i^{K,j}\}_{i=1}^{n_K^j}$  denote the restriction to an element  $K$  of the subset of the basis  $\{\phi_i^j\}_{i=1}^{N^j}$  of  $V_h^j$  supported on  $K$ .

We may now compute  $A$  by summing the contributions from each local cell  $K$ ,

$$\begin{aligned} A_{\iota_K(i)} &= \sum_{K \in \mathcal{T}} \int_K I^c(\phi_{\iota_K(i_1)}^1, \dots, \phi_{\iota_K(i_r)}^r; w_1, \dots, w_n) \, dx \\ &= \sum_{K \in \mathcal{T}} \int_K I^c(\phi_{i_1}^{K,1}, \dots, \phi_{i_r}^{K,r}; w_1, \dots, w_n) \, dx = \sum_{K \in \mathcal{T}} A_i^K, \end{aligned} \quad (2.30)$$

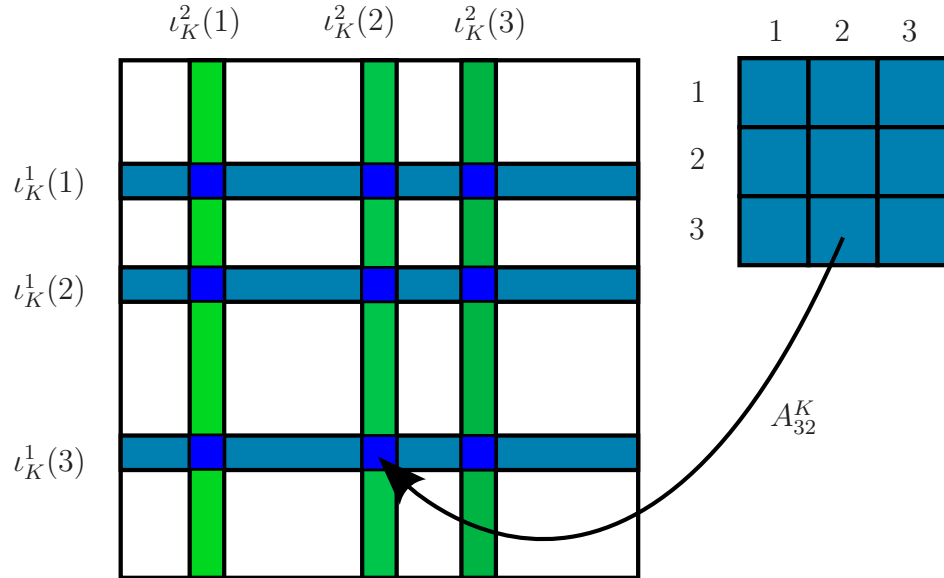


Figure 2.1: Adding the entries of a cell tensor  $A^K$  to the global tensor  $A$  using the local-to-global mapping  $\iota_K$ , illustrated here for a rank two tensor (a matrix).

where  $A^K$  is the local *cell tensor* on cell  $K$  (the “element stiffness matrix”). Similarly, we may sum the local contributions from the exterior and interior facets in the form of local *exterior facet tensors* and *interior facet tensors*.

In Algorithm 1, we present a general algorithm for assembling the contributions from the local cell, exterior facet and interior facet tensors into a global sparse tensor. In Appendix B, we present a sketch of the corresponding implementation in C++ based on UFC. In both cases, we iterate over the local cells, exterior and interior facets. On each local entity, we compute the local tensor and add it to the global tensor as illustrated in Figure 2.1.

## UFC Specification and User Manual 1.0

---

**Algorithm 1** Assembling the global tensor  $A$  from the local contributions on all cells, exterior and interior facets. For assembly over exterior facets,  $K$  refers to the cell  $K \in \mathcal{T}$  incident to the exterior facet  $S$ , and for assembly over interior facets,  $K$  refers to the “macro cell” consisting of the pair of cells  $K^+$  and  $K^-$  incident to the interior facet  $S$ .

---

$A = 0$

*Assemble contributions from all cells*

**for each**  $K \in \mathcal{T}$

**for**  $j = 1, 2, \dots, r$ :

Tabulate the local-to-global mapping  $\iota_K^j$

Let  $K \in \mathcal{T}_k$

Tabulate the cell tensor  $A^K$  for  $I_k^c$

Add  $A_i^K$  to  $A_{\iota_K^1(i_1), \iota_K^2(i_2), \dots, \iota_K^r(i_r)}$  for all  $i \in I_K$

*Assemble contributions from all exterior facets*

**for each**  $S \in \partial_e \mathcal{T}$

**for**  $j = 1, 2, \dots, r$ :

Tabulate the local-to-global mapping  $\iota_K^j$

Let  $S \in \partial_e \mathcal{T}_k$

Tabulate the exterior facet tensor  $A^S$  for  $I_k^e$

Add  $A_i^S$  to  $A_{\iota_K^1(i_1), \iota_K^2(i_2), \dots, \iota_K^r(i_r)}$  for all  $i \in I_K$

*Assemble contributions from all interior facets*

**for each**  $S \in \partial_i \mathcal{T}$

**for**  $j = 1, 2, \dots, r$ :

Tabulate the local-to-global mapping  $\iota_K^j$

Let  $S \in \partial_i \mathcal{T}_k$

Tabulate the interior facet tensor  $A^S$  for  $I_k^i$

Add  $A_i^S$  to  $A_{\iota_K^1(i_1), \iota_K^2(i_2), \dots, \iota_K^r(i_r)}$  for all  $i \in I_K$



# Chapter 3

## Interface specification

### 3.1 A short remark on design

UFC is organized as a minimalistic set of abstract C++ classes representing low-level abstractions of the finite element method. The functions in these classes are mainly of two types: (i) functions returning dimensions, which are typically used to allocate an array, and (ii) functions that fill an array with values.

It is considered the assembly routine's responsibility to allocate and deallocate arrays of proper size. Consider for example the function for evaluating the  $i$ th basis function in the class `ufc::finite_element`:

```
virtual void evaluate_basis(unsigned int i,  
                           double* values,  
                           const double* coordinates,  
                           const cell& c) const = 0;
```

This function assumes that the array `values` has the correct size, which may be obtained by calling the functions `value_rank` and `value_dimension` as described in detail below.

## UFC Specification and User Manual 1.0

---

Thus, the UFC interface is a low-level interface that should be simple to integrate into an existing C++ finite element code, but which is probably not suitable to be exposed as part of an end-user interface.

The UFC interface is defined by a single header file `ufc.h` which defines the central interface class `ufc::form` and a small set of auxiliary interface classes. In addition, a pair of data structures `ufc::mesh` and `ufc::cell` are defined and used for passing data to the interface functions. All functions defined by the UFC interface are *pure virtual*, meaning that all these functions must be overloaded in each implementation of the classes. All but two functions (`init_mesh` and `init_cell`) are `const`, meaning that calling these `const` functions will leave the UFC objects unchanged. Input argument (pointers) are always `const`, while output arguments (pointers) are always non-`const`.

The interface is presented below in the same order as it is defined in the header file `ufc.h`. Thus, the interface is presented bottom-up, starting with the definition of basic data structures and ending with the definition of the main `ufc::form` interface class.

### 3.2 Cell shapes

```
enum shape {interval,  
            triangle,  
            quadrilateral,  
            tetrahedron,  
            hexahedron};
```

This enumeration includes all cell shapes that are covered by the UFC specification, see Chapter 4.



### 3.3 The class `ufc::mesh`

The class `ufc::mesh` defines a data structure containing basic information about an unstructured mesh. It is used for passing a minimal amount of information about the global mesh to UFC functions.

#### 3.3.1 The integer `topological_dimension`

```
unsigned int topological_dimension;
```

The unsigned integer `topological_dimension` holds the topological dimension of the mesh, that is, the topological dimension of the cells of the mesh. For the supported cell shapes defined above, the topological dimensions are as follows: `interval` has dimension one, `triangle` and `quadrilateral` have dimension two, and `tetrahedron` and `hexahedron` have dimension three.

#### 3.3.2 The integer `geometric_dimension`

```
unsigned int geometric_dimension;
```

The unsigned integer `geometric_dimension` holds the geometric dimension of the mesh, that is, the dimension of the coordinates of the mesh vertices. Often, the geometric dimension is equal to the topological dimension, but they may differ. For example, one may have a topologically two-dimensional mesh embedded in three-dimensional space.

#### 3.3.3 The array `num_entities`

```
unsigned int* num_entities;
```

## UFC Specification and User Manual 1.0

---

The array `num_entities` should contain the number of entities within each topological dimension of the mesh (see Chapter 4). The size of the array should be equal to the topological dimension of the mesh plus one.

Thus, for a mesh of tetrahedral cells, `num_entities[0]` should contain the number of vertices, `num_entities[1]` should contain the number of edges (if they are needed, see `ufc::dof_map::needs_mesh_entities` below), `num_entities[2]` should contain the number of faces, and `num_entities[3]` should contain the number of volumes. If `d` is the topological dimension of the mesh, `num_entities[d]` should contain the number of cells or elements.

### 3.4 The class `ufc::cell`

The class `ufc::cell` defines the data structure for a cell in a mesh. Its intended use is not as a building block in a mesh data structure, but merely as a view of specific data for a single cell. It is used to pass cell data to UFC functions with a minimal amount of assumptions on how the computational mesh is represented and stored.

#### 3.4.1 The enum variable `cell_shape`

```
shape cell_shape;
```

The variable `cell_shape` should be set to the corresponding `ufc::shape` for the cell.

#### 3.4.2 The integer `topological_dimension`

```
unsigned int topological_dimension;
```

## UFC Specification and User Manual 1.0

---

The integer `topological_dimension` should be set to the topological dimension of the cell (see `ufc::mesh` above).

### 3.4.3 The integer `geometric_dimension`

```
unsigned int geometric_dimension;
```

The integer `geometric_dimension` should be set to the geometric dimension of the cell (see `ufc::mesh` above).

### 3.4.4 The array `entity_indices`

```
unsigned int** entity_indices;
```

The array `entity_indices` should contain the global indices for all entities of the cell (see Chapter 4). The length of the array `entity_indices` should be equal to the value of the variable `topological_dimension` plus one.

Thus, `entity_indices[0]` should be an array containing the global indices of all the vertices of the cell, `entity_indices[1]` should be an array containing the global indices of all the edges of the cell, etc. The sizes of these arrays are implicitly defined by the cell type.

Note that the entity indices are not always needed for all entities of the cell. Which entities are required is specified by the `ufc::dof_map` class (see `ufc::dof_map::needs_mesh_entities` below).

### 3.4.5 The array `coordinates`

```
double** coordinates;
```

## UFC Specification and User Manual 1.0

---

The array `coordinates` should contain the global coordinates for all vertices of the cell and thus its length should be equal to number of vertices of the cell. The length of the array `coordinates[0]` should be equal to the value of the variable `geometric_dimension` and it should contain the  $x, y, \dots$  coordinates of the first vertex etc.

### 3.5 The class `ufc::function`

The class `ufc::function` is an interface for evaluation of general tensor-valued functions on the cells of a mesh.

#### 3.5.1 The function `evaluate`

```
virtual void evaluate(double* values,
                    const double* coordinates,
                    const cell& c) const = 0;
```

The only function in this class is `evaluate`, which evaluates all the value components of the function at a given point in a given cell of the mesh.

The output of `evaluate` should be written to the array `values`. For a scalar-valued function, a single value should be written to `values[0]`. For general tensor-valued functions, the values should be written in a flattened row-major ordering of the tensor values. Thus, for a function  $f : K \rightarrow \mathbb{R}^{2 \times 2}$  (where  $A = f(x)$  is a  $2 \times 2$  matrix), the array `values` should contain the values  $A_{11}, A_{12}, A_{21}, A_{22}$  in this order.

The input to `evaluate` are the coordinates of a point in a cell and the UFC view of the cell containing that point.

See also the description of `ufc::finite_element::evaluate_dof` below.

### 3.6 The class `ufc::finite_element`

The class `ufc::finite_element` represents a finite element in the classical Ciarlet sense [6], or rather a particular instance of a finite element for a particular choice of nodal basis functions. Thus, a `ufc::finite_element` has functions for accessing the shape of the finite element, the dimension of the polynomial function space, the basis functions of the function space (and their derivatives), and the linear functionals defining the degrees of freedom. In addition, a `ufc::finite_element` provides functionality for interpolation.

#### 3.6.1 The function `signature`

```
virtual const char* signature() const = 0;
```

This function returns a signature string that uniquely identifies the finite element. This can be used to compare whether or not two given `ufc::finite_element` objects are identical.

#### 3.6.2 The function `cell_shape`

```
virtual shape cell_shape() const = 0;
```

This function returns the shape of the cell the finite element is defined on.

#### 3.6.3 The function `space_dimension`

```
virtual unsigned int space_dimension() const = 0;
```

## UFC Specification and User Manual 1.0

---

This function returns the dimension of the local finite element space ( $|V_h^K|$ ), which is equal to the number of basis functions. This should also be equal to the value of `local_dimension()` for the corresponding `ufc::dof_map` (see below).

### 3.6.4 The function `value_rank`

```
virtual unsigned int value_rank() const = 0;
```

A finite element can have general tensor-valued basis functions. The function `value_rank` returns the rank of the value space of the basis functions. For a scalar element, this function should return zero, for vector-valued functions it should return one, for matrix-valued functions it should return two, etc.

### 3.6.5 The function `value_dimension`

```
virtual unsigned int  
value_dimension(unsigned int i) const = 0;
```

This function returns the dimension of the value space of the finite element basis functions for the given axis, where the given axis must be a number between zero and the value rank minus one.

Note that the total size (number of values) of the value space is obtained as the product of `value_dimension(i)` for  $0 \leq i < \text{value\_rank}()$ .

### 3.6.6 The function `evaluate_basis`

```
virtual void evaluate_basis(unsigned int i,  
                           double* values,
```

## UFC Specification and User Manual 1.0

---

```
const double* coordinates,  
const cell& c) const = 0;
```

This function evaluates basis function `i` at the given `coordinates` within the given cell `c`, and stores the values in the array `values`. The size of the output array should be equal to size of the value space (see `value_dimension` above).

### 3.6.7 The function `evaluate_basis_derivatives`

```
virtual void  
evaluate_basis_derivatives(unsigned int i,  
                           unsigned int n,  
                           double* values,  
                           const double* coordinates,  
                           const ufc::cell& c) const = 0;
```

This function evaluates all order `n` derivatives of basis function `i` at the given `coordinates` within the given cell, and stores the values in the array `values`. Derivatives may be obtained up to the polynomial degree of the finite element function space with higher degree derivatives evaluating to zero.

The number of derivatives is given by  $d^n$  where  $d$  is the geometric dimension of the cell. For  $n = 1$ ,  $d = 3$ , the order of the derivatives is naturally  $\partial/\partial x$ ,  $\partial/\partial y$ ,  $\partial/\partial z$ . For  $n = 2$ ,  $d = 3$ , the order of the derivatives is  $\frac{\partial^2}{\partial x \partial x}$ ,  $\frac{\partial^2}{\partial x \partial y}$ ,  $\dots$ ,  $\frac{\partial^2}{\partial z \partial z}$ . Thus, the derivatives are stored in a flattened row-major ordering based on the derivative spatial dimensions.

For tensor-valued basis functions, the ordering of derivatives is row-major based on the value space dimensions followed by the derivative spatial dimensions.

### 3.6.8 The function `evaluate_dof`

```
virtual double evaluate_dof(unsigned int i,  
                           const function& f,  
                           const cell& c) const = 0;
```

This function evaluates and returns the value of the degree of freedom `i` (which is a linear functional) on the given function `f`.

For example, the degrees of freedom for Lagrange finite elements are given by evaluation of `f` at a set of points. Other examples of degrees of freedom include weighted integrals over facets or normal components on facets.

### 3.6.9 The function `interpolate_vertex_values`

```
virtual void  
interpolate_vertex_values(double* vertex_values,  
                         const double* dof_values,  
                         const cell& c) const = 0;
```

This function takes as input the array `dof_values` containing the expansion coefficients for some function in the nodal basis and computes the values of that function at the vertices of the given cell, storing those values in the array `vertex_values`. If the function is tensor-valued, the values are stored in the array `vertex_values` row-major on the list of vertices followed by the row-major ordering of the tensor values as described above.

### 3.6.10 The function `num_sub_elements`

```
virtual unsigned int num_sub_elements() const = 0;
```



## UFC Specification and User Manual 1.0

---

This function returns the number of subelements for a nested (mixed) element. For simple elements (non-nested), this function should return one.

A nested element is an element that is defined from a set of elements by taking the direct sum (tensor product) of the polynomial spaces of those elements. For example, the basis functions  $\psi_1, \psi_2, \dots, \psi_m$  of a vector-valued Lagrange element may be constructed from a scalar Lagrange element by repeating the basis functions  $\phi_1, \phi_2, \dots, \phi_n$  of the scalar element and padding with zeros:  $\psi_1 = (\phi_1, 0), \psi_2 = (\phi_2, 0), \dots, \psi_n = (\phi_n, 0), \psi_{n+1} = (0, \phi_1), \psi_{n+2} = (0, \phi_2), \dots$

Finite elements may be nested at arbitrary depth. For example, a mixed Taylor–Hood element may be created by combining a vector-valued quadratic Lagrange finite element with a scalar linear Lagrange finite element, and the vector-valued element may in turn be created by combining a set of scalar quadratic Lagrange elements.

### 3.6.11 The function `create_sub_element`

```
virtual finite_element*  
create_sub_element(unsigned int i) const = 0;
```

This factory function constructs a `ufc::finite_element` object for subelement `i`. The argument `i` must be an integer between zero and the number of subelements (`num_sub_elements`) minus one. If the element is simple (non-nested), this function creates a copy of the finite element itself. The caller is responsible for deleting the returned object.

## 3.7 The class `ufc::dof_map`

This class represents the local-to-global mapping of degrees of freedom (dofs), or rather one particular instance of such a mapping (there are many possible local-to-global mappings) as defined in Equation (2.27). The most central

## UFC Specification and User Manual 1.0

---

function of this class is `tabulate_dofs`, which tabulates the local-to-global mapping from the degree of freedom indices on a local cell to a global vector of degree of freedom indices.

### 3.7.1 The function signature

```
virtual const char* signature() const = 0;
```

This function returns a signature string that uniquely identifies the dofmap. This can be used to compare whether or not two given `ufc::dof_map` objects are identical. (This may be used to optimize the assembly of forms by caching previously computed dofmaps.)

### 3.7.2 The function needs\_mesh\_entities

```
virtual bool needs_mesh_entities(unsigned int d) const = 0;
```

This function returns true if the `ufc::dof_map` requires mesh entities of topological dimension `d` to be available in `ufc::cell` arguments. Thus, if this function returns false for a given value of `d`, then the array `entity_indices[d]` of the `ufc::cell` data structure will not be used during calls to `ufc::dof_map` functions. In those cases, the array `entity_indices[d]` may be set to zero.

This may be used to check which entities must be generated to tabulate the local-to-global mapping. For example, linear Lagrange elements will only need to know the vertices of each cell in the mesh, while quadratic Lagrange elements will also need to know the edges of each cell in the mesh.

### 3.7.3 The function init\_mesh

```
virtual bool init_mesh(const mesh& mesh) = 0;
```

## UFC Specification and User Manual 1.0

---

This function initializes the dofmap for a given mesh. If it returns true, calls to `init_cell` and `init_cell_finalize` are required to complete the initialization. The function `global_dimension` may only be called when the initialization is complete.

### 3.7.4 The function `init_cell`

```
virtual void init_cell(const mesh& m,  
                      const cell& c) = 0;
```

For `ufc::dof_map` objects where `init_mesh` returns true, this function must be called for each cell in the mesh to initialize the dofmap.

### 3.7.5 The function `init_cell_finalize`

```
virtual void init_cell_finalize() = 0;
```

For `ufc::dof_map` objects where `init_mesh` returns true, this function must be called after `init_cell` is called for each cell in the mesh to complete initialization of the dofmap.

### 3.7.6 The function `global_dimension`

```
virtual unsigned int global_dimension() const = 0;
```

This function returns the dimension of the global finite element space on the mesh that the `ufc::dof_map` has been initialized for. The result of calling this function before the initialization is complete is undefined.

### 3.7.7 The function `local_dimension`

```
virtual unsigned int local_dimension() const = 0;
```

This function returns the dimension of the local finite element space on a single cell.

### 3.7.8 The function `num_facet_dofs`

```
virtual unsigned int num_facet_dofs() const = 0;
```

This function returns the number of dofs tied to a single facet of a cell. This is the number of dofs that should be set if a Dirichlet boundary condition is applied to a single facet. It is assumed that all cells of the mesh have the same number of degrees of freedom on each of their facets.

### 3.7.9 The function `tabulate_dofs`

```
virtual void tabulate_dofs(unsigned int* dofs,  
                           const mesh& m,  
                           const cell& c) const = 0;
```

This function tabulates the global dof indices corresponding to each dof on the given cell. The size of the output array `dofs` should be equal to the value returned by `local_dimension()`.

### 3.7.10 The function `tabulate_facet_dofs`

```
virtual void  
tabulate_facet_dofs(unsigned int* dofs,
```

## UFC Specification and User Manual 1.0

---

```
unsigned int facet) const = 0;
```

This function tabulates the local dof indices tied to a given local facet. The size of the output array `dofs` should be equal to the value returned by `num_facet_dofs()`.

### 3.7.11 The function `tabulate_coordinates`

```
virtual void tabulate_coordinates(double** coordinates,  
                                 const cell& c) const = 0;
```

This function tabulates the coordinates for each dof on the given cell. For Lagrange elements, this function will tabulate a set of points on the given cell such that the dofs of the finite element are given by evaluation at those points.

For elements that do not have a direct relationship between coordinates and dofs, an attempt should be made at a sensible implementation of this function. For example, if a dof is defined as the integral over a facet, the midpoint of the facet can be used. If no other choice makes sense, the midpoint of the cell can be used as a last resort. This function must thus be used with care if non-Lagrangian elements are used.

The size of the output array `coordinates` should be equal to the value returned by `local_dimension()` and the size of each subarray `coordinates[0]`, `coordinates[1]` etc should be equal to the geometric dimension of the mesh.

### 3.7.12 The function `num_sub_dof_maps`

```
virtual unsigned int num_sub_dof_maps() const = 0;
```

## UFC Specification and User Manual 1.0

---

This function returns the number of sub-dofmaps for a nested (mixed) element. For a discussion on the subelement concept, see the documentation of the function `ufc::finite_element::num_sub_elements`. For simple elements (non-nested), this function should return one.

### 3.7.13 The function `create_sub_dof_map`

```
virtual dof_map* create_sub_dof_map(unsigned int i) const = 0;
```

This factory function constructs a `ufc::dof_map` object for subelement `i`. The argument `i` must be a number between zero and the number of sub-dofmaps (`num_sub_dof_maps`) minus one. If the dofmap is simple (non-nested), this function creates a copy of the dofmap itself. The caller is responsible for deleting the returned object.

## 3.8 The integral classes

As described in Section 2, and in particular Equation (2.14), the global sparse tensor (the “stiffness matrix”) representing a given form (differential operator) may be assembled by summing the contributions from the local cells, exterior facets and interior facets of the mesh.

These contributions are represented in the UFC interface by the classes `cell_integral`, `exterior_facet_integral` and `interior_facet_integral`. Each of these three integral classes has a single function `tabulate_tensor` which may be called to compute the corresponding local contribution (cell tensor, exterior facet tensor or interior facet tensor).

## 3.9 The class `ufc::cell_integral`

The class `ufc::cell_integral` represents the integral of a form over a local cell in a finite element mesh. It has a single function `tabulate_tensor` which may be called to tabulate the values of the cell tensor for a given cell.

### 3.9.1 The function `tabulate_tensor`

```
virtual void tabulate_tensor(double* A,  
                             const double * const * w,  
                             const cell& c) const = 0;
```

This function tabulates the values of the cell tensor for a form into the given array `A`. The size of this array should be equal to the product of the local space dimensions for the set of finite element function spaces corresponding to the arguments of the form. For example, when computing the matrix for a bilinear form defined on piecewise linear scalar elements on triangles, the space dimension of the local finite element is three and so the size of the array `A` should be  $3 \times 3 = 9$ .

The array `w` should contain the expansion coefficients for all *coefficients* of the form in the finite element nodal basis for each corresponding function space. Thus, the size of the array `w` should be equal to the number of coefficients  $n$ , and the size of each each array `w[0]`, `w[1]` etc should be equal to the space dimension of the corresponding local finite element space.

## 3.10 The class `ufc::exterior_facet_integral`

The class `ufc::exterior_facet_integral` represents the integral of a form over a local exterior facet (boundary facet) in a finite element mesh. It has a single function `tabulate_tensor` which may be called to tabulate the values of the exterior facet tensor for a given facet.

### 3.10.1 The function `tabulate_tensor`

```
virtual void tabulate_tensor(double* A,  
                             const double * const * w,  
                             const cell& c,  
                             unsigned int facet) const = 0;
```

The arrays `A` and `w` have the same function and should have the same sizes as described in the documentation for `cell_integral::tabulate_tensor`. Thus, the values of the exterior facet integral will be tabulated into the array `A` and the nodal basis expansions of all coefficients should be provided in the array `w`.

The additional argument `facet` should specify the local number of the facet with respect to its (single) incident cell. Thus, when the facet is an edge of a triangle, the argument `facet` should be an integer between zero and two (0, 1, 2) and when the facet is a facet of a tetrahedron, the argument `facet` should be an integer between zero and three (0, 1, 2, 3).

## 3.11 The class `ufc::interior_facet_integral`

The class `ufc::interior_facet_integral` represents the integral of a form over a local interior facet in a finite element mesh. It has a single function `tabulate_tensor` which may be called to tabulate the values of the interior facet tensor for a given facet.

### 3.11.1 The function `tabulate_tensor`

```
virtual void tabulate_tensor(double* A,  
                             const double * const * w,  
                             const cell& c0,  
                             const cell& c1,
```



## UFC Specification and User Manual 1.0

---

```
unsigned int facet0,  
unsigned int facet1) const = 0;
```

Just as for the `cell_integral` and `exterior_facet_integral` classes, the `tabulate_tensor` function for the class `interior_facet_integral` tabulates the values of the local (interior facet) tensor into the array `A`, given the nodal basis expansions of the form coefficients in the array `w`. However, the interior facet tensor contains contributions from the two incident cells of an interior facet and thus the dimensions of these arrays are different.

On each interior facet, the two incident (neighboring) cells form a “macro cell” consisting of the total set of local basis functions on the two cells. The set of basis functions on the macro element is obtained by extending the basis functions on each of the two cells by zero to the macro cell. Thus, the space dimension of the finite element function space on the macro element is twice the size of the finite element function space on a single cell. The ordering of basis functions and degrees of freedom on the macro cell is obtained by first enumerating the basis functions and degrees of freedom on one of the two cells and then the basis functions and degrees of freedom on the second cell.

Thus the size of the array `A` should be equal to the product of twice the local space dimensions for the set of finite element function spaces corresponding to the arguments of the form. For example, when computing the matrix for a bilinear form defined on piecewise linear elements on triangles, the space dimension of the local finite element is three and so the size of the array `A` should be  $6 \times 6 = 36$ .

Similarly, the array `w` should contain the expansion coefficients for all *coefficients* of the form in the finite element nodal basis for each corresponding function space on the macro cell. Thus, the size of the array `w` should be equal to the number of coefficients  $n$  and the size of each each array `w[0]`, `w[1]` etc should be equal to twice the space dimension of the corresponding local finite element space.

The additional arguments `facet0` and `facet1` should specify the local number of the facet with respect to its two incident cells. Thus, when the facet is an edge of a triangle, each of these arguments may be an integer between

zero and two (0, 1, 2) and when the facet is a face of a tetrahedron, each of these arguments may be an integer between zero and three (0, 1, 2, 3).

### 3.12 The class `ufc::form`

The `ufc::form` class is the central part of the UFC interface and it represents a form

$$a = a(v_1, \dots, v_r; w_1, \dots, w_n), \quad (3.1)$$

defined on the product space  $V_h^1 \times V_h^2 \times \dots \times V_h^r \times W_h^1 \times W_h^2 \times \dots \times W_h^n$  of two sets  $\{V_h^j\}_{j=1}^r, \{W_h^j\}_{j=1}^n$  of finite element function spaces on a triangulation  $\mathcal{T}$  of a domain  $\Omega \subset \mathbb{R}^d$ .

A `ufc::form` provides functions for accessing the rank  $r$  and number of coefficients  $n$  for a form, and factory functions for creating UFC objects for the corresponding cell integrals, exterior facet integrals, interior facet integrals, and all associated finite elements and dofmaps (local-to-global mappings).

#### 3.12.1 The function signature

```
virtual const char* signature() const = 0;
```

This function returns a signature string that uniquely identifies the form. This can be used to compare whether or not two given `ufc::form` objects are identical.

#### 3.12.2 The function rank

```
virtual unsigned int rank() const = 0;
```

This function returns the rank  $r$  of the global tensor generated by the form (the arity of the form).

### 3.12.3 The function `num_coefficients`

```
virtual unsigned int num_coefficients() const = 0;
```

This function returns the number of coefficients  $n$  for the form. Note that all integral terms of a form must have the same coefficients, even if not all coefficients are present in each term of the form.

### 3.12.4 The function `num_cell_integrals`

```
virtual unsigned int num_cell_integrals() const = 0;
```

This function returns the number of different cell integrals for the form. A form may have an arbitrary number of integrals over disjoint subdomains of the mesh.

### 3.12.5 The function `num_exterior_facet_integrals`

```
virtual unsigned int num_exterior_facet_integrals() const = 0;
```

This function returns the number of different exterior facet integrals for the form. A form may have an arbitrary number of integrals over disjoint subdomains of the mesh boundary.

### 3.12.6 The function `num_interior_facet_integrals`

```
virtual unsigned int num_interior_facet_integrals() const = 0;
```

This function returns the number of different interior facet integrals for the form. A form may have an arbitrary number of integrals over disjoint subsets of the interior facets of the mesh.

### 3.12.7 The function `create_finite_element`

```
virtual finite_element*  
create_finite_element(unsigned int i) const = 0;
```

This factory function constructs a `ufc::finite_element` object for form argument `i`. A form with rank  $r$  and number of coefficients  $n$  has  $r + n$  arguments, so this function returns the finite element object for tensor axis  $i$  if  $i < r$ , or the finite element for coefficient  $i - r$  if  $i \geq r$ . The caller is responsible for deleting the returned object.

### 3.12.8 The function `create_dof_map`

```
virtual dof_map*  
create_dof_map(unsigned int i) const = 0;
```

This factory function constructs a `ufc::dof_map` object for form argument `i`. A form with rank  $r$  and number of coefficients  $n$  has  $r + n$  arguments, so this function returns the dofmap object for tensor axis  $i$  if  $i < r$ , or the dofmap for coefficient  $i - r$  if  $i \geq r$ . The caller is responsible for deleting the returned object.

### 3.12.9 The function `create_cell_integral`

```
virtual cell_integral*  
create_cell_integral(unsigned int i) const = 0;
```

This factory function constructs a `cell_integral` object for cell domain `i`. The caller is responsible for deleting the returned object.

### 3.12.10 The function `create_exterior_facet_integral`

```
virtual exterior_facet_integral*  
create_exterior_facet_integral(unsigned int i) const = 0;
```

This factory function constructs an `exterior_facet_integral` object for exterior facet domain `i`. The caller is responsible for deleting the returned object.

### 3.12.11 The function `create_interior_facet_integral`

```
virtual interior_facet_integral*  
create_interior_facet_integral(unsigned int i) const = 0;
```

This factory function constructs an `interior_facet_integral` object for interior facet domain `i`. The caller is responsible for deleting the returned object.



# Chapter 4

## Reference cells

The following five reference cells are covered by the UFC specification: the reference *interval*, the reference *triangle*, the reference *quadrilateral*, the reference *tetrahedron* and the reference *hexahedron*.

Reference cell	Dimension	#Vertices	#Facets
The reference interval	1	2	2
The reference triangle	2	3	3
The reference quadrilateral	2	4	4
The reference tetrahedron	3	4	4
The reference hexahedron	3	8	6

Table 4.1: Reference cells covered by the UFC specification.

The UFC specification assumes that each cell in a finite element mesh is always isomorphic to one of the reference cells.

## 4.1 The reference interval

The reference interval is shown in Figure 4.1 and is defined by its two vertices with coordinates as specified in Table 4.2.

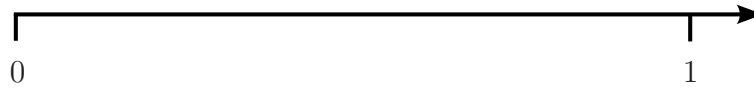


Figure 4.1: The reference interval.

Vertex	Coordinate
$v_0$	$x = 0$
$v_1$	$x = 1$

Table 4.2: Vertex coordinates of the reference interval.



## 4.2 The reference triangle

The reference triangle is shown in Figure 4.2 and is defined by its three vertices with coordinates as specified in Table 4.3.

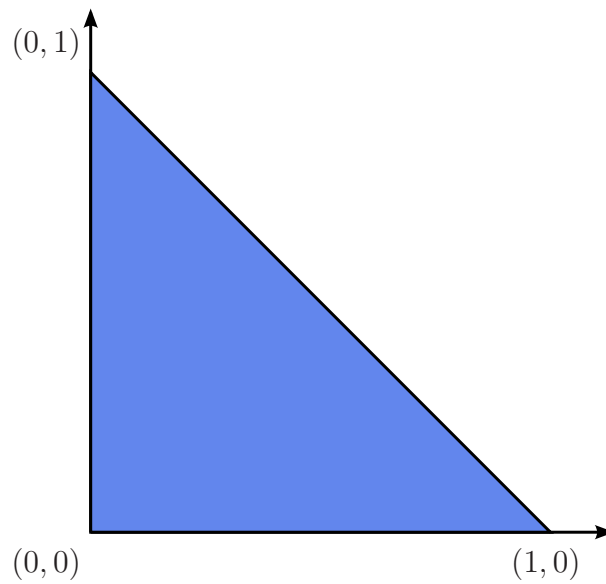


Figure 4.2: The reference triangle.

Vertex	Coordinate
$v_0$	$x = (0, 0)$
$v_1$	$x = (1, 0)$
$v_2$	$x = (0, 1)$

Table 4.3: Vertex coordinates of the reference triangle.

### 4.3 The reference quadrilateral

The reference quadrilateral is shown in Figure 4.3 and is defined by its four vertices with coordinates as specified in Table 4.4.

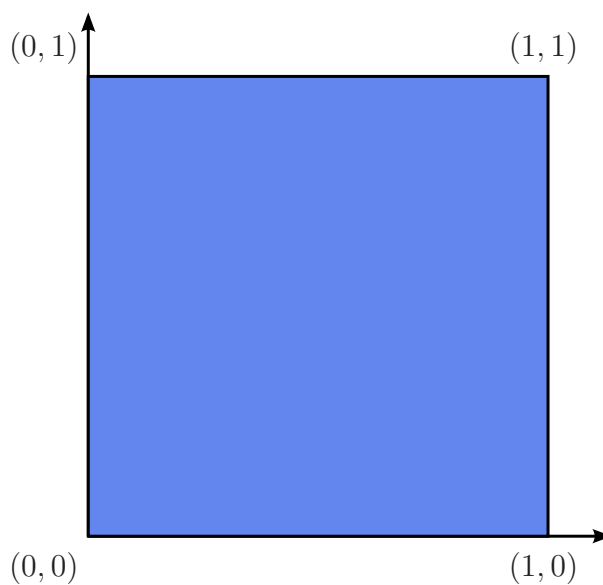


Figure 4.3: The reference quadrilateral.

Vertex	Coordinate
$v_0$	$x = (0, 0)$
$v_1$	$x = (1, 0)$
$v_2$	$x = (1, 1)$
$v_3$	$x = (0, 1)$

Table 4.4: Vertex coordinates of the reference quadrilateral.

## 4.4 The reference tetrahedron

The reference tetrahedron is shown in Figure 4.4 and is defined by its four vertices with coordinates as specified in Table 4.5.

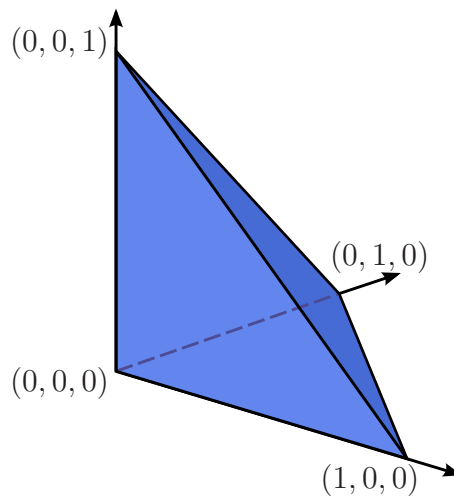


Figure 4.4: The reference tetrahedron.

Vertex	Coordinate
$v_0$	$x = (0, 0, 0)$
$v_1$	$x = (1, 0, 0)$
$v_2$	$x = (0, 1, 0)$
$v_3$	$x = (0, 0, 1)$

Table 4.5: Vertex coordinates of the reference tetrahedron.

## 4.5 The reference hexahedron

The reference hexahedron is shown in Figure 4.5 and is defined by its eight vertices with coordinates as specified in Table 4.6.

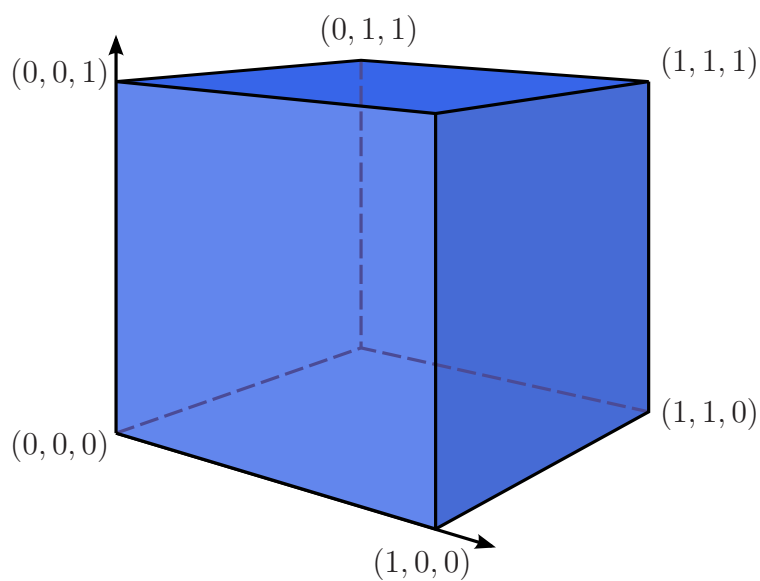


Figure 4.5: The reference hexahedron.

Vertex	Coordinate	Vertex	Coordinate
$v_0$	$x = (0, 0, 0)$	$v_4$	$x = (0, 0, 1)$
$v_1$	$x = (1, 0, 0)$	$v_5$	$x = (1, 0, 1)$
$v_2$	$x = (1, 1, 0)$	$v_6$	$x = (1, 1, 1)$
$v_3$	$x = (0, 1, 0)$	$v_7$	$x = (0, 1, 1)$

Table 4.6: Vertex coordinates of the reference hexahedron.

# Chapter 5

## Numbering of mesh entities

The UFC specification dictates a certain numbering of the vertices, edges etc. of the cells of a finite element mesh. First, an *ad hoc* numbering is picked for the vertices of each cell. Then, the remaining entities are ordered based on a simple rule, as described in detail below.

### 5.1 Basic concepts

The topological entities of a cell (or mesh) are referred to as *mesh entities*. A mesh entity can be identified by a pair  $(d, i)$ , where  $d$  is the topological dimension of the mesh entity and  $i$  is a unique index of the mesh entity. Mesh entities are numbered within each topological dimension from 0 to  $n_d - 1$ , where  $n_d$  is the number of mesh entities of topological dimension  $d$ .

For convenience, mesh entities of topological dimension 0 are referred to as *vertices*, entities of dimension 1 as *edges*, entities of dimension 2 as *faces*, entities of *codimension* 1 as *facets* and entities of codimension 0 as *cells*. These concepts are summarized in Table 5.1.

Thus, the vertices of a tetrahedron are identified as  $v_0 = (0, 0)$ ,  $v_1 = (0, 1)$  and  $v_2 = (0, 2)$ , the edges are  $e_0 = (1, 0)$ ,  $e_1 = (1, 1)$ ,  $e_2 = (1, 2)$ ,  $e_3 = (1, 3)$ ,

$e_4 = (1, 4)$  and  $e_5 = (1, 5)$ , the faces (facets) are  $f_0 = (2, 0)$ ,  $f_1 = (2, 1)$ ,  $f_2 = (2, 2)$  and  $f_3 = (2, 3)$ , and the cell itself is  $c_0 = (3, 0)$ .

Entity	Dimension	Codimension
Vertex	0	–
Edge	1	–
Face	2	–
Facet	–	1
Cell	–	0

Table 5.1: Named mesh entities.

## 5.2 Numbering of vertices

For simplicial cells (intervals, triangles and tetrahedra) of a finite element mesh, the vertices are numbered locally based on the corresponding global vertex numbers. In particular, a tuple of increasing local vertex numbers corresponds to a tuple of increasing global vertex numbers. This is illustrated in Figure 5.1 for a mesh consisting of two triangles.

For non-simplicial cells (quadrilaterals and hexahedra), the numbering is arbitrary, as long as each cell is isomorphic to the corresponding reference cell by matching each vertex with the corresponding vertex in the reference cell. This is illustrated in Figure 5.2 for a mesh consisting of two quadrilaterals.

## 5.3 Numbering of other mesh entities

When the vertices have been numbered, the remaining mesh entities are numbered within each topological dimension based on a *lexicographical ordering* of the corresponding ordered tuples of *non-incident vertices*.

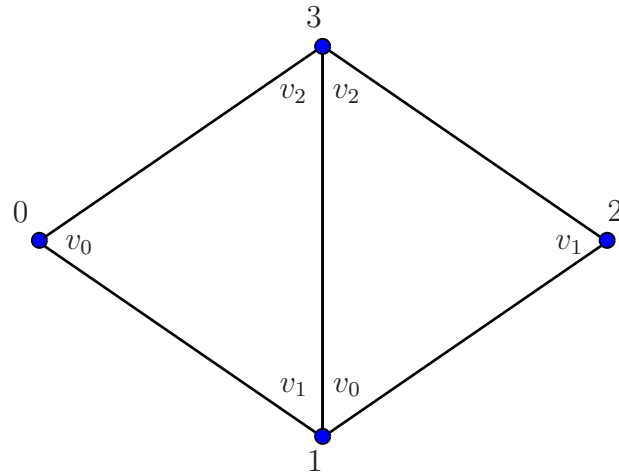


Figure 5.1: The vertices of a simplicial mesh are numbered locally based on the corresponding global vertex numbers.

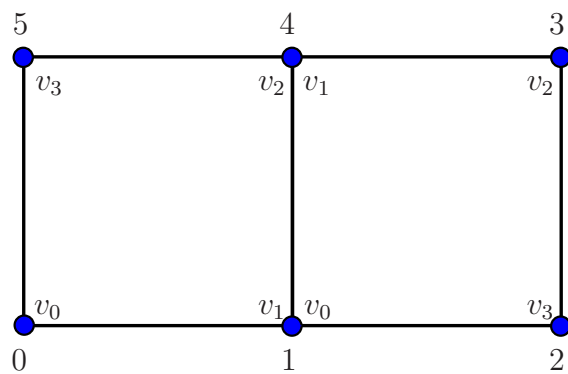


Figure 5.2: The local numbering of vertices of a non-simplicial mesh is arbitrary, as long as each cell is isomorphic to the reference cell by matching each vertex to the corresponding vertex of the reference cell.

As an illustration, consider the numbering of edges (the mesh entities of topological dimension one) on the reference triangle in Figure 5.3. To number the edges of the reference triangle, we identify for each edge the corresponding non-incident vertices. For each edge, there is only one such vertex (the vertex opposite to the edge). We thus identify the three edges in the reference triangle with the tuples  $(v_0)$ ,  $(v_1)$  and  $(v_2)$ . The first of these is edge  $e_0$  between vertices  $v_1$  and  $v_2$  opposite to vertex  $v_0$ , the second is edge  $e_1$  between vertices  $v_0$  and  $v_2$  opposite to vertex  $v_1$ , and the third is edge  $e_2$  between vertices  $v_0$  and  $v_1$  opposite to vertex  $v_2$ .

Similarly, we identify the six edges of the reference tetrahedron with the corresponding non-incident tuples  $(v_0, v_1)$ ,  $(v_0, v_2)$ ,  $(v_0, v_3)$ ,  $(v_1, v_2)$ ,  $(v_1, v_3)$  and  $(v_2, v_3)$ . The first of these is edge  $e_0$  between vertices  $v_2$  and  $v_3$  opposite to vertices  $v_0$  and  $v_1$  as shown in Figure 5.4.

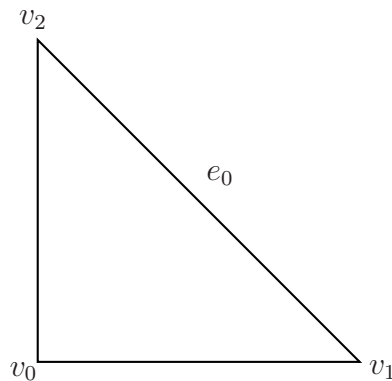


Figure 5.3: Mesh entities are ordered based on a lexicographical ordering of the corresponding ordered tuples of non-incident vertices. The first edge  $e_0$  is non-incident to vertex  $v_0$ .

### 5.3.1 Relative ordering

The relative ordering of mesh entities with respect to other incident mesh entities follows by sorting the entities by their (global) indices. Thus, the pair of vertices incident to the first edge  $e_0$  of a triangular cell is  $(v_1, v_2)$ , not  $(v_2, v_1)$ . Similarly, the first face  $f_0$  of a tetrahedral cell is incident to vertices  $(v_1, v_2, v_3)$ .



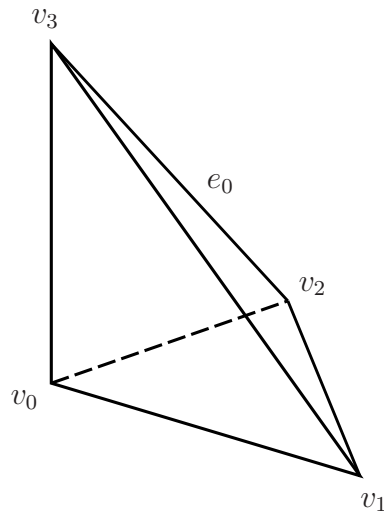


Figure 5.4: Mesh entities are ordered based on a lexicographical ordering of the corresponding ordered tuples of non-incident vertices. The first edge  $e_0$  is non-incident to vertices  $v_0$  and  $v_1$ .

For simplicial cells, the relative ordering in combination with the convention of numbering the vertices locally based on global vertex indices means that two incident cells will always agree on the orientation of incident subsimplices. Thus, two incident triangles will agree on the orientation of the common edge and two incident tetrahedra will agree on the orientation of the common edge(s) and the orientation of the common face (if any). This is illustrated in Figure 5.5 for two incident triangles sharing a common edge.

### 5.3.2 Limitations

The UFC specification is only concerned with the ordering of mesh entities with respect to entities of larger topological dimension. In other words, the UFC specification is only concerned with the ordering of incidence relations of the class  $d - d'$  where  $d > d'$ . For example, the UFC specification is not concerned with the ordering of incidence relations of the class  $0 - 1$ , that is, the ordering of edges incident to vertices.

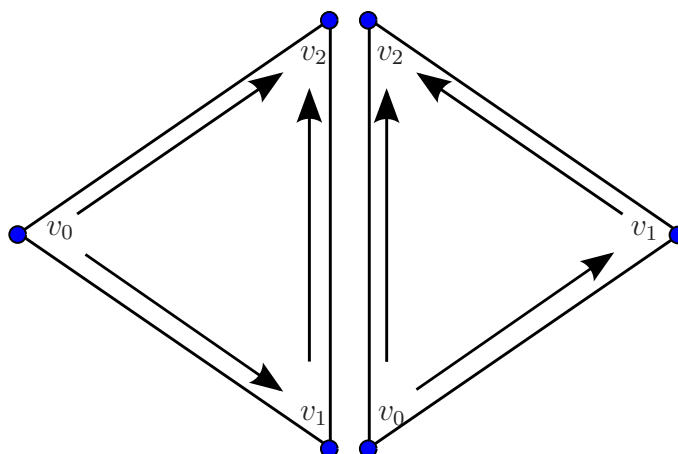


Figure 5.5: Two incident triangles will always agree on the orientation of the common edge.

## 5.4 Numbering schemes for reference cells

The numbering scheme is demonstrated below for cells isomorphic to each of the five reference cells.

### 5.4.1 Numbering for intervals

Entity	Incident vertices	Non-incident vertices
$v_0 = (0, 0)$	$(v_0)$	$(v_1)$
$v_1 = (0, 1)$	$(v_1)$	$(v_0)$
$c_0 = (1, 0)$	$(v_0, v_1)$	$\emptyset$

Table 5.2: Numbering of mesh entities on intervals.

### 5.4.2 Numbering for triangular cells

Entity	Incident vertices	Non-incident vertices
$v_0 = (0, 0)$	$(v_0)$	$(v_1, v_2)$
$v_1 = (0, 1)$	$(v_1)$	$(v_0, v_2)$
$v_2 = (0, 2)$	$(v_2)$	$(v_0, v_1)$
$e_0 = (1, 0)$	$(v_1, v_2)$	$(v_0)$
$e_1 = (1, 1)$	$(v_0, v_2)$	$(v_1)$
$e_2 = (1, 2)$	$(v_0, v_1)$	$(v_2)$
$c_0 = (2, 0)$	$(v_0, v_1, v_2)$	$\emptyset$

Table 5.3: Numbering of mesh entities on triangular cells.

### 5.4.3 Numbering for quadrilateral cells

Entity	Incident vertices	Non-incident vertices
$v_0 = (0, 0)$	$(v_0)$	$(v_1, v_2, v_3)$
$v_1 = (0, 1)$	$(v_1)$	$(v_0, v_2, v_3)$
$v_2 = (0, 2)$	$(v_2)$	$(v_0, v_1, v_3)$
$v_3 = (0, 3)$	$(v_3)$	$(v_0, v_1, v_2)$
$e_0 = (1, 0)$	$(v_2, v_3)$	$(v_0, v_1)$
$e_1 = (1, 1)$	$(v_1, v_2)$	$(v_0, v_3)$
$e_2 = (1, 2)$	$(v_0, v_3)$	$(v_1, v_2)$
$e_3 = (1, 3)$	$(v_0, v_1)$	$(v_2, v_3)$
$c_0 = (2, 0)$	$(v_0, v_1, v_2, v_3)$	$\emptyset$

Table 5.4: Numbering of mesh entities on quadrilateral cells.

### 5.4.4 Numbering for tetrahedral cells

Entity	Incident vertices	Non-incident vertices
$v_0 = (0, 0)$	$(v_0)$	$(v_1, v_2, v_3)$
$v_1 = (0, 1)$	$(v_1)$	$(v_0, v_2, v_3)$
$v_2 = (0, 2)$	$(v_2)$	$(v_0, v_1, v_3)$
$v_3 = (0, 3)$	$(v_3)$	$(v_0, v_1, v_2)$
$e_0 = (1, 0)$	$(v_2, v_3)$	$(v_0, v_1)$
$e_1 = (1, 1)$	$(v_1, v_3)$	$(v_0, v_2)$
$e_2 = (1, 2)$	$(v_1, v_2)$	$(v_0, v_3)$
$e_3 = (1, 3)$	$(v_0, v_3)$	$(v_1, v_2)$
$e_4 = (1, 4)$	$(v_0, v_2)$	$(v_1, v_3)$
$e_5 = (1, 5)$	$(v_0, v_1)$	$(v_2, v_3)$
$f_0 = (2, 0)$	$(v_1, v_2, v_3)$	$(v_0)$
$f_1 = (2, 1)$	$(v_0, v_2, v_3)$	$(v_1)$
$f_2 = (2, 2)$	$(v_0, v_1, v_3)$	$(v_2)$
$f_3 = (2, 3)$	$(v_0, v_1, v_2)$	$(v_3)$
$c_0 = (3, 0)$	$(v_0, v_1, v_2, v_3)$	$\emptyset$

Table 5.5: Numbering of mesh entities on tetrahedral cells.

### 5.4.5 Numbering for hexahedral cells

Entity	Incident vertices	Non-incident vertices
$v_0 = (0, 0)$	$(v_0)$	$(v_1, v_2, v_3, v_4, v_5, v_6, v_7)$
$v_1 = (0, 1)$	$(v_1)$	$(v_0, v_2, v_3, v_4, v_5, v_6, v_7)$
$v_2 = (0, 2)$	$(v_2)$	$(v_0, v_1, v_3, v_4, v_5, v_6, v_7)$
$v_3 = (0, 3)$	$(v_3)$	$(v_0, v_1, v_2, v_4, v_5, v_6, v_7)$
$v_4 = (0, 4)$	$(v_4)$	$(v_0, v_1, v_2, v_3, v_5, v_6, v_7)$
$v_5 = (0, 5)$	$(v_5)$	$(v_0, v_1, v_2, v_3, v_4, v_6, v_7)$
$v_6 = (0, 6)$	$(v_6)$	$(v_0, v_1, v_2, v_3, v_4, v_5, v_7)$
$v_7 = (0, 7)$	$(v_7)$	$(v_0, v_1, v_2, v_3, v_4, v_5, v_6)$
$e_0 = (1, 0)$	$(v_6, v_7)$	$(v_0, v_1, v_2, v_3, v_4, v_5)$
$e_1 = (1, 1)$	$(v_5, v_6)$	$(v_0, v_1, v_2, v_3, v_4, v_7)$
$e_2 = (1, 2)$	$(v_4, v_7)$	$(v_0, v_1, v_2, v_3, v_5, v_6)$
$e_3 = (1, 3)$	$(v_4, v_5)$	$(v_0, v_1, v_2, v_3, v_6, v_7)$
$e_4 = (1, 4)$	$(v_3, v_7)$	$(v_0, v_1, v_2, v_4, v_5, v_6)$
$e_5 = (1, 5)$	$(v_2, v_6)$	$(v_0, v_1, v_3, v_4, v_5, v_7)$
$e_6 = (1, 6)$	$(v_2, v_3)$	$(v_0, v_1, v_4, v_5, v_6, v_7)$
$e_7 = (1, 7)$	$(v_1, v_5)$	$(v_0, v_2, v_3, v_4, v_6, v_7)$
$e_8 = (1, 8)$	$(v_1, v_2)$	$(v_0, v_3, v_4, v_5, v_6, v_7)$
$e_9 = (1, 9)$	$(v_0, v_4)$	$(v_1, v_2, v_3, v_5, v_6, v_7)$
$e_{10} = (1, 10)$	$(v_0, v_3)$	$(v_1, v_2, v_4, v_5, v_6, v_7)$
$e_{11} = (1, 11)$	$(v_0, v_1)$	$(v_2, v_3, v_4, v_5, v_6, v_7)$
$f_0 = (2, 0)$	$(v_4, v_5, v_6, v_7)$	$(v_0, v_1, v_2, v_3)$
$f_1 = (2, 1)$	$(v_2, v_3, v_6, v_7)$	$(v_0, v_1, v_4, v_5)$
$f_2 = (2, 2)$	$(v_1, v_2, v_5, v_6)$	$(v_0, v_3, v_4, v_7)$
$f_3 = (2, 3)$	$(v_0, v_3, v_4, v_7)$	$(v_1, v_2, v_5, v_6)$
$f_4 = (2, 4)$	$(v_0, v_1, v_4, v_5)$	$(v_2, v_3, v_6, v_7)$
$f_5 = (2, 5)$	$(v_0, v_1, v_2, v_3)$	$(v_4, v_5, v_6, v_7)$
$c_0 = (3, 0)$	$(v_0, v_1, v_2, v_3, v_4, v_5, v_6, v_7)$	$\emptyset$

Table 5.6: Numbering of mesh entities on hexahedral cells.



# Bibliography

- [1] *Trilinos*. URL: <http://software.sandia.gov/trilinos/>.
- [2] M. ALNÆS AND K.-A. MARDAL, *SyFi*, 2007. URL: <http://www.fenics.org/syfi/>.
- [3] S. BALAY, K. BUSCHELMAN, W. D. GROPP, D. KAUSHIK, M. G. KNEPLEY, L. C. MCINNES, B. F. SMITH, AND H. ZHANG, *PETSc*, 2006. URL: <http://www.mcs.anl.gov/petsc/>.
- [4] W. BANGERTH, R. HARTMANN, AND G. KANSCHAT, *deal.II Differential Equations Analysis Library*, 2006. URL: <http://www.dealii.org/>.
- [5] A. M. BRUASET, H. P. LANGTANGEN, ET AL., *Diffpack*, 2006. URL: <http://www.diffpack.com/>.
- [6] P. G. CIARLET, *The Finite Element Method for Elliptic Problems*, North-Holland, Amsterdam, New York, Oxford, 1978.
- [7] J. HOFFMAN, J. JANSSON, C. JOHNSON, M. G. KNEPLEY, R. C. KIRBY, A. LOGG, L. R. SCOTT, AND G. N. WELLS, *FEniCS*, 2006. URL: <http://www.fenics.org/>.
- [8] J. HOFFMAN, J. JANSSON, A. LOGG, AND G. N. WELLS, *DOLFIN*, 2006. URL: <http://www.fenics.org/dolfin/>.
- [9] T. J. R. HUGHES, *The Finite Element Method: Linear Static and Dynamic Finite Element Analysis*, Prentice-Hall, 1987.

## UFC Specification and User Manual 1.0

---

- [10] R. C. KIRBY, M. G. KNEPLEY, A. LOGG, AND L. R. SCOTT, *Optimizing the evaluation of finite element matrices*, SIAM J. Sci. Comput., 27 (2005), pp. 741–758.
- [11] R. C. KIRBY AND A. LOGG, *A compiler for variational forms*, ACM Transactions on Mathematical Software, 32 (2006), pp. 417–444.
- [12] ———, *Efficient compilation of a class of variational forms*, ACM Transactions on Mathematical Software, 33 (2007).
- [13] R. C. KIRBY, A. LOGG, L. R. SCOTT, AND A. R. TERREL, *Topological optimization of the evaluation of finite element matrices*, SIAM J. Sci. Comput., 28 (2006), pp. 224–240.
- [14] H. P. LANGTANGEN, *Computational Partial Differential Equations – Numerical Methods and Diffpack Programming*, Lecture Notes in Computational Science and Engineering, Springer, 1999.
- [15] A. LOGG, *FFC*, 2007. URL: <http://www.fenics.org/ffc/>.
- [16] K. LONG, *Sundance*, 2006. URL: <http://software.sandia.gov/sundance/>.
- [17] O. C. ZIENKIEWICZ, R. L. TAYLOR, AND J. Z. ZHU, *The Finite Element Method — Its Basis and Fundamentals*, 6th edition, Elsevier, 2005, first published in 1967.



# Appendix A

## C++ Interface

Below follows a verbatim copy of the complete UFC interface which is specified in the header file `ufc.h`.

```
// This is UFC (Unified Form-assembly Code) v. 1.0.
// This code is released into the public domain.
//
// The FEniCS Project (http://www.fenics.org/) 2006-2007.

#ifndef __UFC_H
#define __UFC_H

#define UFC_VERSION_MAJOR 1
#define UFC_VERSION_MINOR 0

const char UFC_VERSION[] = "1.0";

namespace ufc
{
    /// Valid cell shapes
    enum shape {interval, triangle, quadrilateral, tetrahedron, hexahedron};

    /// This class defines the data structure for a finite element mesh.
    class mesh
    {
    public:
        /// Constructor
        mesh(): topological_dimension(0), geometric_dimension(0), num_entities(0) {}

        /// Destructor
    };
};
```

## UFC Specification and User Manual 1.0

---

```
virtual ~mesh() {}

/// Topological dimension of the mesh
unsigned int topological_dimension;

/// Geometric dimension of the mesh
unsigned int geometric_dimension;

/// Array of the global number of entities of each topological dimension
unsigned int* num_entities;

};

/// This class defines the data structure for a cell in a mesh.

class cell
{
public:

    /// Constructor
    cell(): cell_shape(interval),
            topological_dimension(0), geometric_dimension(0),
            entity_indices(0), coordinates(0) {}

    /// Destructor
    virtual ~cell() {}

    /// Shape of the cell
    shape cell_shape;

    /// Topological dimension of the mesh
    unsigned int topological_dimension;

    /// Geometric dimension of the mesh
    unsigned int geometric_dimension;

    /// Array of global indices for the mesh entities of the cell
    unsigned int** entity_indices;

    /// Array of coordinates for the vertices of the cell
    double** coordinates;

};

/// This class defines the interface for a general tensor-valued function.

class function
{
public:

    /// Constructor
    function() {}

    /// Destructor
    virtual ~function() {}

    /// Evaluate function at given point in cell
```

## UFC Specification and User Manual 1.0

---

```
    virtual void evaluate(double* values,
                        const double* coordinates,
                        const cell& c) const = 0;

};

/// This class defines the interface for a finite element.

class finite_element
{
public:

    /// Constructor
    finite_element() {}

    /// Destructor
    virtual ~finite_element() {}

    /// Return a string identifying the finite element
    virtual const char* signature() const = 0;

    /// Return the cell shape
    virtual shape cell_shape() const = 0;

    /// Return the dimension of the finite element function space
    virtual unsigned int space_dimension() const = 0;

    /// Return the rank of the value space
    virtual unsigned int value_rank() const = 0;

    /// Return the dimension of the value space for axis i
    virtual unsigned int value_dimension(unsigned int i) const = 0;

    /// Evaluate basis function i at given point in cell
    virtual void evaluate_basis(unsigned int i,
                              double* values,
                              const double* coordinates,
                              const cell& c) const = 0;

    /// Evaluate order n derivatives of basis function i at given point in cell
    virtual void evaluate_basis_derivatives(unsigned int i,
                                           unsigned int n,
                                           double* values,
                                           const double* coordinates,
                                           const ufc::cell& c) const = 0;

    /// Evaluate linear functional for dof i on the function f
    virtual double evaluate_dof(unsigned int i,
                              const function& f,
                              const cell& c) const = 0;

    /// Interpolate vertex values from dof values
    virtual void interpolate_vertex_values(double* vertex_values,
                                          const double* dof_values,
                                          const cell& c) const = 0;

    /// Return the number of sub elements (for a mixed element)
```

## UFC Specification and User Manual 1.0

---

```
virtual unsigned int num_sub_elements() const = 0;

// Create a new finite element for sub element i (for a mixed element)
virtual finite_element* create_sub_element(unsigned int i) const = 0;

};

// This class defines the interface for a local-to-global mapping of
// degrees of freedom (dofs).

class dof_map
{
public:

// Constructor
dof_map() {}

// Destructor
virtual ~dof_map() {}

// Return a string identifying the dof map
virtual const char* signature() const = 0;

// Return true iff mesh entities of topological dimension d are needed
virtual bool needs_mesh_entities(unsigned int d) const = 0;

// Initialize dof map for mesh (return true iff init_cell() is needed)
virtual bool init_mesh(const mesh& mesh) = 0;

// Initialize dof map for given cell
virtual void init_cell(const mesh& m,
                      const cell& c) = 0;

// Finish initialization of dof map for cells
virtual void init_cell_finalize() = 0;

// Return the dimension of the global finite element function space
virtual unsigned int global_dimension() const = 0;

// Return the dimension of the local finite element function space
virtual unsigned int local_dimension() const = 0;

// Return the number of dofs on each cell facet
virtual unsigned int num_facet_dofs() const = 0;

// Tabulate the local-to-global mapping of dofs on a cell
virtual void tabulate_dofs(unsigned int* dofs,
                          const mesh& m,
                          const cell& c) const = 0;

// Tabulate the local-to-local mapping from facet dofs to cell dofs
virtual void tabulate_facet_dofs(unsigned int* dofs,
                                unsigned int facet) const = 0;

// Tabulate the coordinates of all dofs on a cell
virtual void tabulate_coordinates(double** coordinates,
                                 const cell& c) const = 0;
```

## UFC Specification and User Manual 1.0

---

```
    /// Return the number of sub dof maps (for a mixed element)
    virtual unsigned int num_sub_dof_maps() const = 0;

    /// Create a new dof_map for sub dof map i (for a mixed element)
    virtual dof_map* create_sub_dof_map(unsigned int i) const = 0;
};

/// This class defines the interface for the tabulation of the cell
/// tensor corresponding to the local contribution to a form from
/// the integral over a cell.

class cell_integral
{
public:

    /// Constructor
    cell_integral() {}

    /// Destructor
    virtual ~cell_integral() {}

    /// Tabulate the tensor for the contribution from a local cell
    virtual void tabulate_tensor(double* A,
                                const double * const * w,
                                const cell& c) const = 0;
};

/// This class defines the interface for the tabulation of the
/// exterior facet tensor corresponding to the local contribution to
/// a form from the integral over an exterior facet.

class exterior_facet_integral
{
public:

    /// Constructor
    exterior_facet_integral() {}

    /// Destructor
    virtual ~exterior_facet_integral() {}

    /// Tabulate the tensor for the contribution from a local exterior facet
    virtual void tabulate_tensor(double* A,
                                const double * const * w,
                                const cell& c,
                                unsigned int facet) const = 0;
};

/// This class defines the interface for the tabulation of the
/// interior facet tensor corresponding to the local contribution to
/// a form from the integral over an interior facet.

class interior_facet_integral
```

## UFC Specification and User Manual 1.0

---

```
{
public:

    /// Constructor
    interior_facet_integral() {}

    /// Destructor
    virtual ~interior_facet_integral() {}

    /// Tabulate the tensor for the contribution from a local interior facet
    virtual void tabulate_tensor(double* A,
                                const double * const * w,
                                const cell& c0,
                                const cell& c1,
                                unsigned int facet0,
                                unsigned int facet1) const = 0;

};

/// This class defines the interface for the assembly of the global
/// tensor corresponding to a form with r + n arguments, that is, a
/// mapping
///
/// a : V1 x V2 x ... Vr x W1 x W2 x ... x Wn -> R
///
/// with arguments v1, v2, ..., vr, w1, w2, ..., wn. The rank r
/// global tensor A is defined by
///
/// A = a(V1, V2, ..., Vr, w1, w2, ..., wn),
///
/// where each argument Vj represents the application to the
/// sequence of basis functions of Vj and w1, w2, ..., wn are given
/// fixed functions (coefficients).

class form
{
public:

    /// Constructor
    form() {}

    /// Destructor
    virtual ~form() {}

    /// Return a string identifying the form
    virtual const char* signature() const = 0;

    /// Return the rank of the global tensor (r)
    virtual unsigned int rank() const = 0;

    /// Return the number of coefficients (n)
    virtual unsigned int num_coefficients() const = 0;

    /// Return the number of cell integrals
    virtual unsigned int num_cell_integrals() const = 0;

    /// Return the number of exterior facet integrals
```

## UFC Specification and User Manual 1.0

---

```
virtual unsigned int num_exterior_facet_integrals() const = 0;

// Return the number of interior facet integrals
virtual unsigned int num_interior_facet_integrals() const = 0;

// Create a new finite element for argument function i
virtual finite_element* create_finite_element(unsigned int i) const = 0;

// Create a new dof map for argument function i
virtual dof_map* create_dof_map(unsigned int i) const = 0;

// Create a new cell integral on sub domain i
virtual cell_integral* create_cell_integral(unsigned int i) const = 0;

// Create a new exterior facet integral on sub domain i
virtual exterior_facet_integral*
create_exterior_facet_integral(unsigned int i) const = 0;

// Create a new interior facet integral on sub domain i
virtual interior_facet_integral*
create_interior_facet_integral(unsigned int i) const = 0;

};
}
#endif
```





## Appendix B

### A basic UFC-based assembler

Below, we include a sketch of a UFC-based implementation of the assembly of the global tensor  $A$  by summing the local contributions from all cells. The contributions from all exterior and interior facets may be computed similarly.

The implementation is incomplete and system specific details such as interaction with mesh and linear algebra libraries have been omitted.<sup>1</sup>

```
void assemble(..., ufc::form& form, ...)
{
    ...

    // Initialize mesh data structure
    ufc::mesh mesh;
    mesh.num_entities = new unsigned int[...];
    ...

    // Initialize cell data structure
    ufc::cell cell;
    cell.entity_indices = new unsigned int[...];
    cell.coordinates = new double[...];
    ...

    // Create cell integrals
    ufc::cell_integral** cell_integrals;
    cell_integrals = new ufc::cell_integral*[form.num_cell_integrals()];
}
```

---

<sup>1</sup>For an example of a complete implementation of a UFC-based assembler, we refer to the source code of DOLFIN [8], in particular class `Assembler` as implemented in `Assembler.cpp`.

## UFC Specification and User Manual 1.0

---

```
for (unsigned int i = 0; i < form.num_cell_integrals(); i++)
    cell_integrals[i] = form.create_cell_integral(i);

// Create dofmaps
ufc::dof_maps** dof_maps;
dof_maps = new ufc::dof_map*[form.rank() + form.num_coefficients()];
for (unsigned int i = 0; i < form.rank() + form.num_coefficients(); i++)
{
    dof_maps[i] = form.create_dof_map(i);

    // Initialize dofmap
    if (dof_maps[i]->init_mesh(mesh))
    {
        // Iterate over cells
        for (...)
        {
            // Update cell data structure to current cell
            cell.entity_indices[...] = ...
            cell.coordinates[...] = ...
            ...

            // Initialize dofmap for cell
            dof_maps[i]->init_cell(mesh, cell);
        }

        dof_map.init_cell_finalize();
    }
}

// Initialize array of values for the cell tensor
unsigned int size = 1;
for (unsigned int i = 0; i < form.rank(); i++)
    size *= dof_maps[i]->local_dimension();
double* AK = new double[size];

// Initialize array of local to global dofmaps
unsigned int** dofs = new unsigned int*[form.rank()];
for (unsigned int i = 0; i < form.rank(); i++)
    dofs[i] = new unsigned int[dof_maps[i]->local_dimension()];

// Initialize array of coefficient values
double** w = new double*[form.num_coefficients()];
for (unsigned int i = 0; i < form.num_coefficients(); i++)
    w[i] = new double[dof_maps[form.rank() + i]->local_dimension()];

// Iterate over cells
for (...)
{
    // Get number of subdomain for current cell
    const unsigned int sub_domain = ...

    // Update cell data structure to current cell
    cell.entity_indices[...] = ...
    cell.coordinates[...] = ...
    ...

    // Interpolate coefficients (library specific so omitted here)
```

## UFC Specification and User Manual 1.0

---

```
...

// Tabulate dofs for each dimension
for (unsigned int i = 0; i < ufc.form.rank(); i++)
    dof_maps[i]->tabulate_dofs(dofs[i], mesh, cell);

// Tabulate cell tensor
cell_integrals[sub_domain]->tabulate_tensor(AK, w, cell);

// Add entries to global tensor (library specific so omitted here)
...
}

// Delete data structures
delete [] mesh.num_entities;
...
}
```



# Appendix C

## Complete UFC code for Poisson's equation

In this section, a simple example is given of UFC code generated by the form compilers FFC [15, 10, 13, 11, 12] and SyFi [2] for Poisson's equation. The code presented below is generated for the bilinear form of Poisson's equation for standard continuous piecewise linear Lagrange finite elements on a two-dimensional domain  $\Omega$ ,

$$a(v, u) = \int_{\Omega} \nabla v \cdot \nabla u \, dx. \quad (\text{C.1})$$

Although FFC and SyFi are two different form compilers, with very different approaches to code generation, both generate code that conforms to the UFC specification and may thus be used interchangeably within any UFC-based system.

In the generated code, we have omitted the two functions `evaluate_basis` and `evaluate_basis_derivatives`<sup>1</sup> to save space.

---

<sup>1</sup>For FFC, this may be done by using the compiler flags `-fno-evaluate_basis` and `-fno-evaluate_basis_derivatives`.

### C.1 Code generated by FFC

```
// This code conforms with the UFC specification version 1.0
// and was automatically generated by FFC version 0.4.0.

#ifndef __POISSON_H
#define __POISSON_H

#include <cmath>
#include <stdexcept>
#include <ufc.h>

/// This class defines the interface for a finite element.

class PoissonBilinearForm_finite_element_0: public ufc::finite_element
{
public:

    /// Constructor
    PoissonBilinearForm_finite_element_0() : ufc::finite_element()
    {
        // Do nothing
    }

    /// Destructor
    virtual ~PoissonBilinearForm_finite_element_0()
    {
        // Do nothing
    }

    /// Return a string identifying the finite element
    virtual const char* signature() const
    {
        return "Lagrange finite element of degree 1 on a triangle";
    }

    /// Return the cell shape
    virtual ufc::shape cell_shape() const
    {
        return ufc::triangle;
    }

    /// Return the dimension of the finite element function space
    virtual unsigned int space_dimension() const
    {
        return 3;
    }

    /// Return the rank of the value space
    virtual unsigned int value_rank() const
    {
        return 0;
    }

    /// Return the dimension of the value space for axis i

```

## UFC Specification and User Manual 1.0

---

```
virtual unsigned int value_dimension(unsigned int i) const
{
    return 1;
}

/// Evaluate basis function i at given point in cell
virtual void evaluate_basis(unsigned int i,
                           double* values,
                           const double* coordinates,
                           const ufc::cell& c) const
{
    // Not generated (compiled with -fno-evaluate_basis)
}

/// Evaluate order n derivatives of basis function i at given point in cell
virtual void evaluate_basis_derivatives(unsigned int i,
                                       unsigned int n,
                                       double* values,
                                       const double* coordinates,
                                       const ufc::cell& c) const
{
    // Not generated (compiled with -fno-evaluate_basis_derivatives)
}

/// Evaluate linear functional for dof i on the function f
virtual double evaluate_dof(unsigned int i,
                            const ufc::function& f,
                            const ufc::cell& c) const
{
    double values[1];
    double coordinates[2];

    // Nodal coordinates on reference cell
    static double X[3][2] = {{0, 0}, {1, 0}, {0, 1}};

    // Components for each dof
    static unsigned int components[3] = {0, 0, 0};

    // Extract vertex coordinates
    const double * const * x = c.coordinates;

    // Evaluate basis functions for affine mapping
    const double w0 = 1.0 - X[i][0] - X[i][1];
    const double w1 = X[i][0];
    const double w2 = X[i][1];

    // Compute affine mapping x = F(X)
    coordinates[0] = w0*x[0][0] + w1*x[1][0] + w2*x[2][0];
    coordinates[1] = w0*x[0][1] + w1*x[1][1] + w2*x[2][1];

    // Evaluate function at coordinates
    f.evaluate(values, coordinates, c);

    // Pick component for evaluation
    return values[components[i]];
}
```

## UFC Specification and User Manual 1.0

---

```
/// Interpolate vertex values from dof values
virtual void interpolate_vertex_values(double* vertex_values,
                                     const double* dof_values,
                                     const ufc::cell& c) const
{
    // Evaluate at vertices and use affine mapping
    vertex_values[0] = dof_values[0];
    vertex_values[1] = dof_values[1];
    vertex_values[2] = dof_values[2];
}

/// Return the number of sub elements (for a mixed element)
virtual unsigned int num_sub_elements() const
{
    return 1;
}

/// Create a new finite element for sub element i (for a mixed element)
virtual ufc::finite_element* create_sub_element(unsigned int i) const
{
    return new PoissonBilinearForm_finite_element_0();
}
};

/// This class defines the interface for a finite element.
class PoissonBilinearForm_finite_element_1: public ufc::finite_element
{
public:
    /// Constructor
    PoissonBilinearForm_finite_element_1() : ufc::finite_element()
    {
        // Do nothing
    }

    /// Destructor
    virtual ~PoissonBilinearForm_finite_element_1()
    {
        // Do nothing
    }

    /// Return a string identifying the finite element
    virtual const char* signature() const
    {
        return "Lagrange finite element of degree 1 on a triangle";
    }

    /// Return the cell shape
    virtual ufc::shape cell_shape() const
    {
        return ufc::triangle;
    }

    /// Return the dimension of the finite element function space
    virtual unsigned int space_dimension() const
```



## UFC Specification and User Manual 1.0

---

```
{
    return 3;
}

/// Return the rank of the value space
virtual unsigned int value_rank() const
{
    return 0;
}

/// Return the dimension of the value space for axis i
virtual unsigned int value_dimension(unsigned int i) const
{
    return 1;
}

/// Evaluate basis function i at given point in cell
virtual void evaluate_basis(unsigned int i,
                           double* values,
                           const double* coordinates,
                           const ufc::cell& c) const
{
    // Not generated (compiled with -fno-evaluate_basis)
}

/// Evaluate order n derivatives of basis function i at given point in cell
virtual void evaluate_basis_derivatives(unsigned int i,
                                       unsigned int n,
                                       double* values,
                                       const double* coordinates,
                                       const ufc::cell& c) const
{
    // Not generated (compiled with -fno-evaluate_basis_derivatives)
}

/// Evaluate linear functional for dof i on the function f
virtual double evaluate_dof(unsigned int i,
                           const ufc::function& f,
                           const ufc::cell& c) const
{
    double values[1];
    double coordinates[2];

    // Nodal coordinates on reference cell
    static double X[3][2] = {{0, 0}, {1, 0}, {0, 1}};

    // Components for each dof
    static unsigned int components[3] = {0, 0, 0};

    // Extract vertex coordinates
    const double * const * x = c.coordinates;

    // Evaluate basis functions for affine mapping
    const double w0 = 1.0 - X[i][0] - X[i][1];
    const double w1 = X[i][0];
    const double w2 = X[i][1];
}
```

## UFC Specification and User Manual 1.0

---

```
// Compute affine mapping x = F(X)
coordinates[0] = w0*x[0][0] + w1*x[1][0] + w2*x[2][0];
coordinates[1] = w0*x[0][1] + w1*x[1][1] + w2*x[2][1];

// Evaluate function at coordinates
f.evaluate(values, coordinates, c);

// Pick component for evaluation
return values[components[i]];
}

/// Interpolate vertex values from dof values
virtual void interpolate_vertex_values(double* vertex_values,
                                     const double* dof_values,
                                     const ufc::cell& c) const
{
    // Evaluate at vertices and use affine mapping
    vertex_values[0] = dof_values[0];
    vertex_values[1] = dof_values[1];
    vertex_values[2] = dof_values[2];
}

/// Return the number of sub elements (for a mixed element)
virtual unsigned int num_sub_elements() const
{
    return 1;
}

/// Create a new finite element for sub element i (for a mixed element)
virtual ufc::finite_element* create_sub_element(unsigned int i) const
{
    return new PoissonBilinearForm_finite_element_1();
}
};

/// This class defines the interface for a local-to-global mapping of
/// degrees of freedom (dofs).

class PoissonBilinearForm_dof_map_0: public ufc::dof_map
{
private:
    unsigned int __global_dimension;

public:
    /// Constructor
    PoissonBilinearForm_dof_map_0() : ufc::dof_map()
    {
        __global_dimension = 0;
    }

    /// Destructor
    virtual ~PoissonBilinearForm_dof_map_0()
    {
        // Do nothing
    }
};
```

## UFC Specification and User Manual 1.0

---

```
}

/// Return a string identifying the dof map
virtual const char* signature() const
{
    return "FFC dof map for Lagrange finite element of degree 1 on a triangle";
}

/// Return true iff mesh entities of topological dimension d are needed
virtual bool needs_mesh_entities(unsigned int d) const
{
    switch ( d )
    {
    case 0:
        return true;
        break;
    case 1:
        return false;
        break;
    case 2:
        return false;
        break;
    }
    return false;
}

/// Initialize dof map for mesh (return true iff init_cell() is needed)
virtual bool init_mesh(const ufc::mesh& m)
{
    __global_dimension = m.num_entities[0];
    return false;
}

/// Initialize dof map for given cell
virtual void init_cell(const ufc::mesh& m,
                     const ufc::cell& c)
{
    // Do nothing
}

/// Finish initialization of dof map for cells
virtual void init_cell_finalize()
{
    // Do nothing
}

/// Return the dimension of the global finite element function space
virtual unsigned int global_dimension() const
{
    return __global_dimension;
}

/// Return the dimension of the local finite element function space
virtual unsigned int local_dimension() const
{
    return 3;
}
```

## UFC Specification and User Manual 1.0

---

```
/// Return the number of dofs on each cell facet
virtual unsigned int num_facet_dofs() const
{
    return 2;
}

/// Tabulate the local-to-global mapping of dofs on a cell
virtual void tabulate_dofs(unsigned int* dofs,
                          const ufc::mesh& m,
                          const ufc::cell& c) const
{
    dofs[0] = c.entity_indices[0][0];
    dofs[1] = c.entity_indices[0][1];
    dofs[2] = c.entity_indices[0][2];
}

/// Tabulate the local-to-local mapping from facet dofs to cell dofs
virtual void tabulate_facet_dofs(unsigned int* dofs,
                                unsigned int facet) const
{
    switch ( facet )
    {
        {
            case 0:
                dofs[0] = 1;
                dofs[1] = 2;
                break;
            case 1:
                dofs[0] = 0;
                dofs[1] = 2;
                break;
            case 2:
                dofs[0] = 0;
                dofs[1] = 1;
                break;
        }
    }
}

/// Tabulate the coordinates of all dofs on a cell
virtual void tabulate_coordinates(double** coordinates,
                                 const ufc::cell& c) const
{
    const double * const * x = c.coordinates;
    coordinates[0][0] = x[0][0];
    coordinates[0][1] = x[0][1];
    coordinates[1][0] = x[1][0];
    coordinates[1][1] = x[1][1];
    coordinates[2][0] = x[2][0];
    coordinates[2][1] = x[2][1];
}

/// Return the number of sub dof maps (for a mixed element)
virtual unsigned int num_sub_dof_maps() const
{
    return 1;
}
```

## UFC Specification and User Manual 1.0

---

```
    /// Create a new dof_map for sub dof map i (for a mixed element)
    virtual ufc::dof_map* create_sub_dof_map(unsigned int i) const
    {
        return new PoissonBilinearForm_dof_map_0();
    }
};

/// This class defines the interface for a local-to-global mapping of
/// degrees of freedom (dofs).

class PoissonBilinearForm_dof_map_1: public ufc::dof_map
{
private:
    unsigned int __global_dimension;

public:
    /// Constructor
    PoissonBilinearForm_dof_map_1() : ufc::dof_map()
    {
        __global_dimension = 0;
    }

    /// Destructor
    virtual ~PoissonBilinearForm_dof_map_1()
    {
        // Do nothing
    }

    /// Return a string identifying the dof map
    virtual const char* signature() const
    {
        return "FFC dof map for Lagrange finite element of degree 1 on a triangle";
    }

    /// Return true iff mesh entities of topological dimension d are needed
    virtual bool needs_mesh_entities(unsigned int d) const
    {
        switch ( d )
        {
            case 0:
                return true;
                break;
            case 1:
                return false;
                break;
            case 2:
                return false;
                break;
        }
        return false;
    }

    /// Initialize dof map for mesh (return true iff init_cell() is needed)
    virtual bool init_mesh(const ufc::mesh& m)
```

## UFC Specification and User Manual 1.0

---

```
{
  __global_dimension = m.num_entities[0];
  return false;
}

/// Initialize dof map for given cell
virtual void init_cell(const ufc::mesh& m,
                      const ufc::cell& c)
{
  // Do nothing
}

/// Finish initialization of dof map for cells
virtual void init_cell_finalize()
{
  // Do nothing
}

/// Return the dimension of the global finite element function space
virtual unsigned int global_dimension() const
{
  return __global_dimension;
}

/// Return the dimension of the local finite element function space
virtual unsigned int local_dimension() const
{
  return 3;
}

/// Return the number of dofs on each cell facet
virtual unsigned int num_facet_dofs() const
{
  return 2;
}

/// Tabulate the local-to-global mapping of dofs on a cell
virtual void tabulate_dofs(unsigned int* dofs,
                          const ufc::mesh& m,
                          const ufc::cell& c) const
{
  dofs[0] = c.entity_indices[0][0];
  dofs[1] = c.entity_indices[0][1];
  dofs[2] = c.entity_indices[0][2];
}

/// Tabulate the local-to-local mapping from facet dofs to cell dofs
virtual void tabulate_facet_dofs(unsigned int* dofs,
                                 unsigned int facet) const
{
  switch ( facet )
  {
  case 0:
    dofs[0] = 1;
    dofs[1] = 2;
    break;
  case 1:
```

## UFC Specification and User Manual 1.0

---

```
    dofs[0] = 0;
    dofs[1] = 2;
    break;
case 2:
    dofs[0] = 0;
    dofs[1] = 1;
    break;
}
}

/// Tabulate the coordinates of all dofs on a cell
virtual void tabulate_coordinates(double** coordinates,
                                const ufc::cell& c) const
{
    const double * const * x = c.coordinates;
    coordinates[0][0] = x[0][0];
    coordinates[0][1] = x[0][1];
    coordinates[1][0] = x[1][0];
    coordinates[1][1] = x[1][1];
    coordinates[2][0] = x[2][0];
    coordinates[2][1] = x[2][1];
}

/// Return the number of sub dof maps (for a mixed element)
virtual unsigned int num_sub_dof_maps() const
{
    return 1;
}

/// Create a new dof_map for sub dof map i (for a mixed element)
virtual ufc::dof_map* create_sub_dof_map(unsigned int i) const
{
    return new PoissonBilinearForm_dof_map_1();
}
};

/// This class defines the interface for the tabulation of the cell
/// tensor corresponding to the local contribution to a form from
/// the integral over a cell.
class PoissonBilinearForm_cell_integral_0: public ufc::cell_integral
{
public:
    /// Constructor
    PoissonBilinearForm_cell_integral_0() : ufc::cell_integral()
    {
        // Do nothing
    }

    /// Destructor
    virtual ~PoissonBilinearForm_cell_integral_0()
    {
        // Do nothing
    }
};
```

## UFC Specification and User Manual 1.0

---

```
/// Tabulate the tensor for the contribution from a local cell
virtual void tabulate_tensor(double* A,
                            const double * const * w,
                            const ufc::cell& c) const
{
  // Extract vertex coordinates
  const double * const * x = c.coordinates;

  // Compute Jacobian of affine map from reference cell
  const double J_00 = x[1][0] - x[0][0];
  const double J_01 = x[2][0] - x[0][0];
  const double J_10 = x[1][1] - x[0][1];
  const double J_11 = x[2][1] - x[0][1];

  // Compute determinant of Jacobian
  double detJ = J_00*J_11 - J_01*J_10;

  // Compute inverse of Jacobian
  const double Jinv_00 = J_11 / detJ;
  const double Jinv_01 = -J_01 / detJ;
  const double Jinv_10 = -J_10 / detJ;
  const double Jinv_11 = J_00 / detJ;

  // Take absolute value of determinant
  detJ = std::abs(detJ);

  // Set scale factor
  const double det = detJ;

  // Compute geometry tensors
  const double G0_0_0 = det*(Jinv_00*Jinv_00 + Jinv_01*Jinv_01);
  const double G0_0_1 = det*(Jinv_00*Jinv_10 + Jinv_01*Jinv_11);
  const double G0_1_0 = det*(Jinv_10*Jinv_00 + Jinv_11*Jinv_01);
  const double G0_1_1 = det*(Jinv_10*Jinv_10 + Jinv_11*Jinv_11);

  // Compute element tensor
  A[0] = 0.5*G0_0_0 + 0.5*G0_0_1 + 0.5*G0_1_0 + 0.5*G0_1_1;
  A[1] = -0.5*G0_0_0 - 0.5*G0_1_0;
  A[2] = -0.5*G0_0_1 - 0.5*G0_1_1;
  A[3] = -0.5*G0_0_0 - 0.5*G0_0_1;
  A[4] = 0.5*G0_0_0;
  A[5] = 0.5*G0_0_1;
  A[6] = -0.5*G0_1_0 - 0.5*G0_1_1;
  A[7] = 0.5*G0_1_0;
  A[8] = 0.5*G0_1_1;
}

};

/// This class defines the interface for the assembly of the global
/// tensor corresponding to a form with r + n arguments, that is, a
/// mapping
///
/// a : V1 x V2 x ... Vr x W1 x W2 x ... x Wn -> R
///
/// with arguments v1, v2, ..., vr, w1, w2, ..., wn. The rank r
/// global tensor A is defined by
```



## UFC Specification and User Manual 1.0

---

```
///
///   A = a(V1, V2, ..., Vr, w1, w2, ..., wn),
///
/// where each argument Vj represents the application to the
/// sequence of basis functions of Vj and w1, w2, ..., wn are given
/// fixed functions (coefficients).

class PoissonBilinearForm: public ufc::form
{
public:

    /// Constructor
    PoissonBilinearForm() : ufc::form()
    {
        // Do nothing
    }

    /// Destructor
    virtual ~PoissonBilinearForm()
    {
        // Do nothing
    }

    /// Return a string identifying the form
    virtual const char* signature() const
    {
        return "(dXa0/dxb0)(dXa1/dxb0) | ((d/dXa0)vi0)*((d/dXa1)vi1)*dX(0)";
    }

    /// Return the rank of the global tensor (r)
    virtual unsigned int rank() const
    {
        return 2;
    }

    /// Return the number of coefficients (n)
    virtual unsigned int num_coefficients() const
    {
        return 0;
    }

    /// Return the number of cell integrals
    virtual unsigned int num_cell_integrals() const
    {
        return 1;
    }

    /// Return the number of exterior facet integrals
    virtual unsigned int num_exterior_facet_integrals() const
    {
        return 0;
    }

    /// Return the number of interior facet integrals
    virtual unsigned int num_interior_facet_integrals() const
    {
        return 0;
    }
};
```

## UFC Specification and User Manual 1.0

---

```
}

// Create a new finite element for argument function i
virtual ufc::finite_element* create_finite_element(unsigned int i) const
{
    switch ( i )
    {
        case 0:
            return new PoissonBilinearForm_finite_element_0();
            break;
        case 1:
            return new PoissonBilinearForm_finite_element_1();
            break;
    }
    return 0;
}

// Create a new dof map for argument function i
virtual ufc::dof_map* create_dof_map(unsigned int i) const
{
    switch ( i )
    {
        case 0:
            return new PoissonBilinearForm_dof_map_0();
            break;
        case 1:
            return new PoissonBilinearForm_dof_map_1();
            break;
    }
    return 0;
}

// Create a new cell integral on sub domain i
virtual ufc::cell_integral* create_cell_integral(unsigned int i) const
{
    return new PoissonBilinearForm_cell_integral_0();
}

// Create a new exterior facet integral on sub domain i
virtual ufc::exterior_facet_integral*
create_exterior_facet_integral(unsigned int i) const
{
    return 0;
}

// Create a new interior facet integral on sub domain i
virtual ufc::interior_facet_integral*
create_interior_facet_integral(unsigned int i) const
{
    return 0;
}
};

#endif
```

### C.2 Code generated by SyFi

In the following we list the complete code for the finite element, the dofmap and the variational form for computing a stiffness matrix based on linear Lagrangian elements in 2D.

The code below is generated for the assembler in PyCC and it therefore includes some PyCC files, since the option `SFC.options.include_from = "pycc"` was used during the code generation. If PyCC is not present, the option `SFC.options.include_from = "syfi"` can be used, and this will result in some additional files that define the numbering scheme.

#### C.2.1 Header file for linear Lagrange element in 2D

```
//
// This code complies with UFC version 1.0, and is generated with SyFi version 0.4.0.
//
// http://www.fenics.org/syfi/
// http://www.fenics.org/ufc/
//

#ifndef __fe_Lagrange_1_2D_H
#define __fe_Lagrange_1_2D_H

#include <stdexcept>
#include <math.h>
#include <ufc.h>
#include <pycc/Functions/Ptv.h>
#include <pycc/Functions/Ptv_tools.h>
#include <pycc/Functions/Dof_Ptv.h>
#include <pycc/Functions/OrderedPtvSet.h>
#include <pycc/Functions/Dof_OrderedPtvSet.h>

namespace pycc
{
    /// This class defines the interface for a finite element.
    class fe_Lagrange_1_2D: public ufc::finite_element
    {
    public:
        /// Constructor
        fe_Lagrange_1_2D();
    };
}
```

## UFC Specification and User Manual 1.0

---

```
/// Destructor
virtual ~fe_Lagrange_1_2D();

/// Return a string identifying the finite element
virtual const char* signature() const;

/// Return the cell shape
virtual ufc::shape cell_shape() const;

/// Return the dimension of the finite element function space
virtual unsigned int space_dimension() const;

/// Return the rank of the value space
virtual unsigned int value_rank() const;

/// Return the dimension of the value space for axis i
virtual unsigned int value_dimension(unsigned int i) const;

/// Evaluate basis function i at given point in cell
virtual void evaluate_basis(unsigned int i,
                           double* values,
                           const double* coordinates,
                           const ufc::cell& c) const;

/// Evaluate order n derivatives of basis function i at given point in cell
virtual void evaluate_basis_derivatives(unsigned int i,
                                       unsigned int n,
                                       double* values,
                                       const double* coordinates,
                                       const ufc::cell& c) const;

/// Evaluate linear functional for dof i on the function f
virtual double evaluate_dof(unsigned int i,
                           const ufc::function& f,
                           const ufc::cell& c) const;

/// Interpolate vertex values from dof values
virtual void interpolate_vertex_values(double* vertex_values,
                                       const double* dof_values,
                                       const ufc::cell& c) const;

/// Return the number of sub elements (for a mixed element)
virtual unsigned int num_sub_elements() const;

/// Create a new finite element for sub element i (for a mixed element)
virtual ufc::finite_element* create_sub_element(unsigned int i) const;
};

} // namespace

#endif
```

### C.2.2 Source file for linear Lagrange element in 2D

```
//
// This code complies with UFC version 1.0, and is generated with SyFi version 0.4.0.
//
// http://www.fenics.org/syfi/
// http://www.fenics.org/ufc/
//

#include <stdexcept>
#include <math.h>
#include <ufc.h>
#include <pycc/Functions/Ptv.h>
#include <pycc/Functions/Ptv_tools.h>
#include <pycc/Functions/Dof_Ptv.h>
#include <pycc/Functions/OrderedPtvSet.h>
#include <pycc/Functions/Dof_OrderedPtvSet.h>
#include "fe_Lagrange_1_2D.h"

namespace pycc
{

    /// Constructor
    fe_Lagrange_1_2D::fe_Lagrange_1_2D() : ufc::finite_element()
    {

    }

    /// Destructor
    fe_Lagrange_1_2D::~~fe_Lagrange_1_2D()
    {

    }

    /// Return a string identifying the finite element
    const char* fe_Lagrange_1_2D::signature() const
    {
        return "fe_Lagrange_1_2D // generated by SyFi";
    }

    /// Return the cell shape
    ufc::shape fe_Lagrange_1_2D::cell_shape() const
    {
        return ufc::triangle;
    }

    /// Return the dimension of the finite element function space
    unsigned int fe_Lagrange_1_2D::space_dimension() const
    {
        return 3;
    }

    /// Return the rank of the value space
```

## UFC Specification and User Manual 1.0

---

```
unsigned int fe_Lagrange_1_2D::value_rank() const
{
    return 0;
}

/// Return the dimension of the value space for axis i
unsigned int fe_Lagrange_1_2D::value_dimension(unsigned int i) const
{
    return 1;
}

/// Evaluate basis function i at given point in cell
void fe_Lagrange_1_2D::evaluate_basis(unsigned int i,
                                     double* values,
                                     const double* coordinates,
                                     const ufc::cell& c) const
{
    const double x = coordinates[0];
    const double y = coordinates[1];
    switch(i)
    {
    case 0:
        values[0] = -x-y+1.0;
        break;
    case 1:
        values[0] = x;
        break;
    case 2:
        values[0] = y;
        break;
    }
}

/// Evaluate order n derivatives of basis function i at given point in cell
void fe_Lagrange_1_2D::evaluate_basis_derivatives(unsigned int i,
                                                  unsigned int n,
                                                  double* values,
                                                  const double* coordinates,
                                                  const ufc::cell& c) const
{
    throw std::runtime_error("gen_evaluate_basis_derivatives not implemented yet.");
}

/// Evaluate linear functional for dof i on the function f
double fe_Lagrange_1_2D::evaluate_dof(unsigned int i,
                                       const ufc::function& f,
                                       const ufc::cell& c) const
{
    // coordinates
    double x0 = c.coordinates[0][0]; double y0 = c.coordinates[0][1];
    double x1 = c.coordinates[1][0]; double y1 = c.coordinates[1][1];
    double x2 = c.coordinates[2][0]; double y2 = c.coordinates[2][1];

    // affine map
    double G00 = x1 - x0;
    double G01 = x2 - x0;
```

## UFC Specification and User Manual 1.0

---

```
double G10 = y1 - y0;
double G11 = y2 - y0;

double v[1];
double x[2];
switch(i)
{
case 0:
x[0] = x0;
x[1] = y0;
break;
case 1:
x[0] = x0+G00;
x[1] = G10+y0;
break;
case 2:
x[0] = G01+x0;
x[1] = y0+G11;
break;
}
f.evaluate(v, x, c);
return v[i % 1];
}

/// Interpolate vertex values from dof values
void fe_Lagrange_1_2D::interpolate_vertex_values(double* vertex_values,
const double* dof_values,
const ufc::cell& c) const
{
vertex_values[0] = dof_values[0];
vertex_values[1] = dof_values[1];
vertex_values[2] = dof_values[2];
}

/// Return the number of sub elements (for a mixed element)
unsigned int fe_Lagrange_1_2D::num_sub_elements() const
{
return 1;
}

/// Create a new finite element for sub element i (for a mixed element)
ufc::finite_element* fe_Lagrange_1_2D::create_sub_element(unsigned int i) const
{
return new fe_Lagrange_1_2D();
}

} // namespace
```

### C.2.3 Header file for the dofmap

```
//
// This code complies with UFC version 1.0, and is generated with SyFi version 0.4.0.
//
// http://www.fenics.org/syfi/
// http://www.fenics.org/ufc/
//

#ifndef __dof_map_Lagrange_1_2D_H
#define __dof_map_Lagrange_1_2D_H

#include <stdexcept>
#include <math.h>
#include <ufc.h>
#include <pycc/Functions/Ptv.h>
#include <pycc/Functions/Ptv_tools.h>
#include <pycc/Functions/Dof_Ptv.h>
#include <pycc/Functions/OrderedPtvSet.h>
#include <pycc/Functions/Dof_OrderedPtvSet.h>

namespace pycc
{
    /// This class defines the interface for a local-to-global mapping of
    /// degrees of freedom (dofs).

    class dof_map_Lagrange_1_2D: public ufc::dof_map
    { public:
        pycc::Dof_Ptv dof;
        unsigned int num_elements;
        unsigned int * loc2glob;

    public:

        /// Constructor
        dof_map_Lagrange_1_2D();

        /// Destructor
        virtual ~dof_map_Lagrange_1_2D();

        /// Return a string identifying the dof map
        virtual const char* signature() const;

        /// Return true iff mesh entities of topological dimension d are needed
        virtual bool needs_mesh_entities(unsigned int d) const;

        /// Initialize dof map for mesh (return true iff init_cell() is needed)
        virtual bool init_mesh(const ufc::mesh& m);

        /// Initialize dof map for given cell
        virtual void init_cell(const ufc::mesh& m,
                               const ufc::cell& c);
    };
}

```



## UFC Specification and User Manual 1.0

---

```
/// Finish initialization of dof map for cells
virtual void init_cell_finalize();

/// Return the dimension of the global finite element function space
virtual unsigned int global_dimension() const;

/// Return the dimension of the local finite element function space
virtual unsigned int local_dimension() const;

/// Return the number of dofs on each cell facet
virtual unsigned int num_facet_dofs() const;

/// Tabulate the local-to-global mapping of dofs on a cell
virtual void tabulate_dofs(unsigned int* dofs,
                          const ufc::mesh& m,
                          const ufc::cell& c) const;

/// Tabulate the local-to-local mapping from facet dofs to cell dofs
virtual void tabulate_facet_dofs(unsigned int* dofs,
                                unsigned int facet) const;

/// Tabulate the coordinates of all dofs on a cell
virtual void tabulate_coordinates(double** coordinates,
                                 const ufc::cell& c) const;

/// Return the number of sub dof maps (for a mixed element)
virtual unsigned int num_sub_dof_maps() const;

/// Create a new dof_map for sub dof map i (for a mixed element)
virtual ufc::dof_map* create_sub_dof_map(unsigned int i) const;
};

} // namespace

#endif
```

### C.2.4 Source file for the dofmap

```
//
// This code complies with UFC version 1.0, and is generated with SyFi version 0.4.0.
//
// http://www.fenics.org/syfi/
// http://www.fenics.org/ufc/
//

#include <stdexcept>
#include <math.h>
#include <ufc.h>
```

## UFC Specification and User Manual 1.0

---

```
#include <pycc/Functions/Ptv.h>
#include <pycc/Functions/Ptv_tools.h>
#include <pycc/Functions/Dof_Ptv.h>
#include <pycc/Functions/OrderedPtvSet.h>
#include <pycc/Functions/Dof_OrderedPtvSet.h>
#include "dof_map_Lagrange_1_2D.h"

namespace pycc
{
    // Constructor
    dof_map_Lagrange_1_2D::dof_map_Lagrange_1_2D() : ufc::dof_map()
    {
        loc2glob = 0;
    }

    // Destructor
    dof_map_Lagrange_1_2D::~dof_map_Lagrange_1_2D()
    {
        if(loc2glob) delete [] loc2glob;
    }

    // Return a string identifying the dof map
    const char* dof_map_Lagrange_1_2D::signature() const
    {
        return "dof_map_Lagrange_1_2D // generated by SyFi";
    }

    // Return true iff mesh entities of topological dimension d are needed
    bool dof_map_Lagrange_1_2D::needs_mesh_entities(unsigned int d) const
    {
        switch(d)
        {
            case 0: return true; // vertices
            case 1: return true; // edges
            case 2: return true; // faces
            case 3: return false; // volumes
        }
        return false; // strange unsupported case or error
    }

    // Initialize dof map for mesh (return true iff init_cell() is needed)
    bool dof_map_Lagrange_1_2D::init_mesh(const ufc::mesh& m)
    {
        int top_dim = 2;
        num_elements = m.num_entities[top_dim];
        return true;
    }

    // Initialize dof map for given cell
    void dof_map_Lagrange_1_2D::init_cell(const ufc::mesh& m,
                                         const ufc::cell& c)
    {
        // coordinates
        double x0 = c.coordinates[0][0]; double y0 = c.coordinates[0][1];
        double x1 = c.coordinates[1][0]; double y1 = c.coordinates[1][1];
    }
}
```

## UFC Specification and User Manual 1.0

---

```
double x2 = c.coordinates[2][0]; double y2 = c.coordinates[2][1];

// affine map
double G00 = x1 - x0;
double G01 = x2 - x0;

double G10 = y1 - y0;
double G11 = y2 - y0;

int element = c.entity_indices[2][0];

double dof0[2] = { x0, y0 };
Ptv pdof0(2, dof0);
dof.insert_dof(element, 0, pdof0);

double dof1[2] = { G00+x0, y0+G10 };
Ptv pdof1(2, dof1);
dof.insert_dof(element, 1, pdof1);

double dof2[2] = { x0+G01, G11+y0 };
Ptv pdof2(2, dof2);
dof.insert_dof(element, 2, pdof2);
}

/// Finish initialization of dof map for cells
void dof_map_Lagrange_1_2D::init_cell_finalize()
{
    loc2glob = new unsigned int[num_elements * local_dimension()];
    dof.build_loc2dof(num_elements, local_dimension(), reinterpret_cast<int*>(loc2glob));
}

/// Return the dimension of the global finite element function space
unsigned int dof_map_Lagrange_1_2D::global_dimension() const
{
    return dof.size();
}

/// Return the dimension of the local finite element function space
unsigned int dof_map_Lagrange_1_2D::local_dimension() const
{
    return 3;
}

/// Return the number of dofs on each cell facet
unsigned int dof_map_Lagrange_1_2D::num_facet_dofs() const
{
    return 2;
}

/// Tabulate the local-to-global mapping of dofs on a cell
void dof_map_Lagrange_1_2D::tabulate_dofs(unsigned int* dofs,
                                         const ufc::mesh& m,
                                         const ufc::cell& c) const
{
    const unsigned int *from_dofs = loc2glob + (3 * c.entity_indices[2][0]);
    memcpy(dofs, from_dofs, sizeof(unsigned int)*3);
}
```

## UFC Specification and User Manual 1.0

---

```
/// Tabulate the local-to-local mapping from facet dofs to cell dofs
void dof_map_Lagrange_1_2D::tabulate_facet_dofs(unsigned int* dofs,
                                              unsigned int facet) const
{
    switch(facet)
    {
    case 0:
        dofs[0] = 1;
        dofs[1] = 2;
        break;
    case 1:
        dofs[0] = 0;
        dofs[1] = 2;
        break;
    case 2:
        dofs[0] = 0;
        dofs[1] = 1;
        break;
    default:
        throw std::runtime_error("Invalid facet number.");
    }
}

/// Tabulate the coordinates of all dofs on a cell
void dof_map_Lagrange_1_2D::tabulate_coordinates(double** coordinates,
                                              const ufc::cell& c) const
{
    // coordinates
    double x0 = c.coordinates[0][0]; double y0 = c.coordinates[0][1];
    double x1 = c.coordinates[1][0]; double y1 = c.coordinates[1][1];
    double x2 = c.coordinates[2][0]; double y2 = c.coordinates[2][1];

    // affine map
    double G00 = x1 - x0;
    double G01 = x2 - x0;

    double G10 = y1 - y0;
    double G11 = y2 - y0;

    coordinates[0][0] = x0;
    coordinates[0][1] = y0;
    coordinates[1][0] = G00+x0;
    coordinates[1][1] = y0+G10;
    coordinates[2][0] = x0+G01;
    coordinates[2][1] = G11+y0;
}

/// Return the number of sub dof maps (for a mixed element)
unsigned int dof_map_Lagrange_1_2D::num_sub_dof_maps() const
{
    return 1;
}

/// Create a new dof_map for sub dof map i (for a mixed element)
ufc::dof_map* dof_map_Lagrange_1_2D::create_sub_dof_map(unsigned int i) const
```

## UFC Specification and User Manual 1.0

---

```
{
    return new dof_map_Lagrange_1_2D();
}

} // namespace
```

### C.2.5 Header file for the stiffness matrix form

```
//
// This code complies with UFC version 1.0, and is generated with SyFi version 0.4.0.
//
// http://www.fenics.org/syfi/
// http://www.fenics.org/ufc/
//

#ifndef __form__stiffness_form__Lagrange_1_2D_H
#define __form__stiffness_form__Lagrange_1_2D_H

#include <stdexcept>
#include <math.h>
#include <ufc.h>
#include <pycc/Functions/Ptv.h>
#include <pycc/Functions/Ptv_tools.h>
#include <pycc/Functions/Dof_Ptv.h>
#include <pycc/Functions/OrderedPtvSet.h>
#include <pycc/Functions/Dof_OrderedPtvSet.h>
#include "dof_map_Lagrange_1_2D.h"
#include "fe_Lagrange_1_2D.h"

namespace pycc
{
    /// This class defines the interface for the assembly of the global
    /// tensor corresponding to a form with  $r + n$  arguments, that is, a
    /// mapping
    ///
    ///  $a : V_1 \times V_2 \times \dots \times V_r \times W_1 \times W_2 \times \dots \times W_n \rightarrow R$ 
    ///
    /// with arguments  $v_1, v_2, \dots, v_r, w_1, w_2, \dots, w_n$ . The rank  $r$ 
    /// global tensor  $A$  is defined by
    ///
    ///  $A = a(V_1, V_2, \dots, V_r, w_1, w_2, \dots, w_n)$ ,
    ///
    /// where each argument  $V_j$  represents the application to the
    /// sequence of basis functions of  $V_j$  and  $w_1, w_2, \dots, w_n$  are given
    /// fixed functions (coefficients).

    class form__stiffness_form__Lagrange_1_2D: public ufc::form
```

## UFC Specification and User Manual 1.0

---

```
{
public:

    /// Constructor
    form_stiffness_form_Lagrange_1_2D();

    /// Destructor
    virtual ~form_stiffness_form_Lagrange_1_2D();

    /// Return a string identifying the form
    virtual const char* signature() const;

    /// Return the rank of the global tensor (r)
    virtual unsigned int rank() const;

    /// Return the number of coefficients (n)
    virtual unsigned int num_coefficients() const;

    /// Return the number of cell integrals
    virtual unsigned int num_cell_integrals() const;

    /// Return the number of exterior facet integrals
    virtual unsigned int num_exterior_facet_integrals() const;

    /// Return the number of interior facet integrals
    virtual unsigned int num_interior_facet_integrals() const;

    /// Create a new finite element for argument function i
    virtual ufc::finite_element* create_finite_element(unsigned int i) const;

    /// Create a new dof map for argument function i
    virtual ufc::dof_map* create_dof_map(unsigned int i) const;

    /// Create a new cell integral on sub domain i
    virtual ufc::cell_integral* create_cell_integral(unsigned int i) const;

    /// Create a new exterior facet integral on sub domain i
    virtual ufc::exterior_facet_integral*
        create_exterior_facet_integral(unsigned int i) const;

    /// Create a new interior facet integral on sub domain i
    virtual ufc::interior_facet_integral*
        create_interior_facet_integral(unsigned int i) const;
};

} // namespace

#endif
```

### C.2.6 Source file for the stiffness matrix form

```
//
// This code complies with UFC version 1.0, and is generated with SyFi version 0.4.0.
//
// http://www.fenics.org/syfi/
// http://www.fenics.org/ufc/
//

#include <stdexcept>
#include <math.h>
#include <ufc.h>
#include <pycc/Functions/Ptv.h>
#include <pycc/Functions/Ptv_tools.h>
#include <pycc/Functions/Dof_Ptv.h>
#include <pycc/Functions/OrderedPtvSet.h>
#include <pycc/Functions/Dof_OrderedPtvSet.h>
#include "dof_map_Lagrange_1_2D.h"
#include "fe_Lagrange_1_2D.h"
#include "form__stiffness_form__Lagrange_1_2D.h"

namespace pycc
{
    /// This class defines the interface for the tabulation of the cell
    /// tensor corresponding to the local contribution to a form from
    /// the integral over a cell.

    class cell_itg__stiffness_form__Lagrange_1_2D: public ufc::cell_integral
    {
    public:

        /// Constructor
        cell_itg__stiffness_form__Lagrange_1_2D();

        /// Destructor
        virtual ~cell_itg__stiffness_form__Lagrange_1_2D();

        /// Tabulate the tensor for the contribution from a local cell
        virtual void tabulate_tensor(double* A,
                                     const double * const * w,
                                     const ufc::cell& c) const;
    };

    /// Constructor
    cell_itg__stiffness_form__Lagrange_1_2D::cell_itg__stiffness_form__Lagrange_1_2D()
    : ufc::cell_integral()
    {
    }

    /// Destructor

```

## UFC Specification and User Manual 1.0

---

```
cell_itg__stiffness_form__Lagrange_1_2D::~cell_itg__stiffness_form__Lagrange_1_2D()
{
}

// Tabulate the tensor for the contribution from a local cell
void cell_itg__stiffness_form__Lagrange_1_2D::tabulate_tensor(double* A,
    const double * const * w,
    const ufc::cell& c) const
{
    // coordinates
    double x0 = c.coordinates[0][0]; double y0 = c.coordinates[0][1];
    double x1 = c.coordinates[1][0]; double y1 = c.coordinates[1][1];
    double x2 = c.coordinates[2][0]; double y2 = c.coordinates[2][1];

    // affine map
    double G00 = x1 - x0;
    double G01 = x2 - x0;

    double G10 = y1 - y0;
    double G11 = y2 - y0;

    double detG_tmp = G00*G11-G01*G10;
    double detG = fabs(detG_tmp);

    double GinvT00 = G11 / detG_tmp;
    double GinvT01 = -G10 / detG_tmp;
    double GinvT10 = -G01 / detG_tmp;
    double GinvT11 = G00 / detG_tmp;

    memset(A, 0, sizeof(double)*9);

    A[3*0 + 0] = detG*((GinvT01*GinvT01)/2.0+(GinvT11*GinvT11)/2.0+GinvT10*GinvT00
        +GinvT01*GinvT11+(GinvT00*GinvT00)/2.0+(GinvT10*GinvT10)/2.0);
    A[3*0 + 1] = detG*(-(GinvT01*GinvT01)/2.0-GinvT10*GinvT00/2.0
        -GinvT01*GinvT11/2.0-(GinvT00*GinvT00)/2.0);
    A[3*0 + 2] = detG*(-(GinvT11*GinvT11)/2.0-GinvT10*GinvT00/2.0
        -GinvT01*GinvT11/2.0-(GinvT10*GinvT10)/2.0);
    A[3*1 + 0] = detG*(-(GinvT01*GinvT01)/2.0-GinvT10*GinvT00/2.0
        -GinvT01*GinvT11/2.0-(GinvT00*GinvT00)/2.0);
    A[3*1 + 1] = detG*((GinvT01*GinvT01)/2.0+(GinvT00*GinvT00)/2.0);
    A[3*1 + 2] = detG*(GinvT10*GinvT00/2.0+GinvT01*GinvT11/2.0);
    A[3*2 + 0] = detG*(-(GinvT11*GinvT11)/2.0-GinvT10*GinvT00/2.0
        -GinvT01*GinvT11/2.0-(GinvT10*GinvT10)/2.0);
    A[3*2 + 1] = detG*(GinvT10*GinvT00/2.0+GinvT01*GinvT11/2.0);
    A[3*2 + 2] = detG*((GinvT11*GinvT11)/2.0+(GinvT10*GinvT10)/2.0);
}

// Constructor
form__stiffness_form__Lagrange_1_2D::form__stiffness_form__Lagrange_1_2D() : ufc::form()
{
}
}
```



## UFC Specification and User Manual 1.0

---

```
/// Destructor
form__stiffness_form__Lagrange_1_2D::~form__stiffness_form__Lagrange_1_2D()
{
}

/// Return a string identifying the form
const char* form__stiffness_form__Lagrange_1_2D::signature() const
{
    return "form__stiffness_form__Lagrange_1_2D // generated by SyFi";
}

/// Return the rank of the global tensor (r)
unsigned int form__stiffness_form__Lagrange_1_2D::rank() const
{
    return 2;
}

/// Return the number of coefficients (n)
unsigned int form__stiffness_form__Lagrange_1_2D::num_coefficients() const
{
    return 0;
}

/// Return the number of cell integrals
unsigned int form__stiffness_form__Lagrange_1_2D::num_cell_integrals() const
{
    return 1;
}

/// Return the number of exterior facet integrals
unsigned int form__stiffness_form__Lagrange_1_2D::num_exterior_facet_integrals() const
{
    return 0;
}

/// Return the number of interior facet integrals
unsigned int form__stiffness_form__Lagrange_1_2D::num_interior_facet_integrals() const
{
    return 0;
}

/// Create a new finite element for argument function i
ufc::finite_element* form__stiffness_form__Lagrange_1_2D::
    create_finite_element(unsigned int i) const
{
    switch(i)
    {
        {
        case 0:
            return new fe_Lagrange_1_2D();
        case 1:
            return new fe_Lagrange_1_2D();
        }
    }
    throw std::runtime_error("Invalid index in create_finite_element()");
}

/// Create a new dof map for argument function i
```

## UFC Specification and User Manual 1.0

---

```
ufc::dof_map* form__stiffness_form__Lagrange_1_2D::create_dof_map(unsigned int i) const
{
    switch(i)
    {
        case 0:
            return new dof_map_Lagrange_1_2D();
        case 1:
            return new dof_map_Lagrange_1_2D();
    }
    throw std::runtime_error("Invalid index in create_dof_map()");
}

/// Create a new cell integral on sub domain i
ufc::cell_integral* form__stiffness_form__Lagrange_1_2D::
    create_cell_integral(unsigned int i) const
{
    return new cell_itg__stiffness_form__Lagrange_1_2D();
}

/// Create a new exterior facet integral on sub domain i
ufc::exterior_facet_integral* form__stiffness_form__Lagrange_1_2D::
    create_exterior_facet_integral(unsigned int i) const
{
    return 0;
}

/// Create a new interior facet integral on sub domain i
ufc::interior_facet_integral* form__stiffness_form__Lagrange_1_2D::
    create_interior_facet_integral(unsigned int i) const
{
    return 0;
}

} // namespace
```

# Appendix D

## Python utilities

The UFC distribution includes a set of Python utilities for generating code that conforms to the UFC specification. These utilities consist of format string templates for C++ header files (.h files), implementation files (.cpp) and combined header and implementation files (.h files containing both the declaration and definition of the UFC functions).

The following format strings are provided:

```
function_combined_{header, implementation, combined}  
finite_element_{header, implementation, combined}  
dof_map_{header, implementation, combined}  
cell_integral_{header, implementation, combined}  
exterior_facet_integral_{header, implementation, combined}  
interior_facet_integral_{header, implementation, combined}  
form_{header, implementation, combined}
```

We demonstrate below how to use the format string `form_combined` together with a dictionary that specifies the code to be inserted into the format string. Typically, a form compiler will first generate the code to be inserted into the dictionary and then in a later stage write the generated code to file in UFC format using the provided format strings.

## UFC Specification and User Manual 1.0

---

```
from ufc import form_combined

code = {}
code["classname"] = "Poisson",
...
code["rank"] = "    return 2;",
code["num_coefficients"] = "    return 0;",
code["num_cell_integrals"] = "    return 1;",
...

print form_combined % code
```

# Appendix E

## Installation

The UFC package consists of two parts, the main part being a single header file called `ufc.h`. In addition, a set of Python utilities to simplify the generation of UFC code is provided.

Questions, bug reports and patches concerning the installation should be directed to the UFC mailing list at the address

```
ufc-dev@fenics.org
```

UFC must currently be installed directly from source, but Debian (Ubuntu) packages will be available in the future, for UFC and other **FENICS** components.

### E.1 Dependencies and requirements

The installation system is based on distutils, and should work on any system with a standard Python installation.

### E.1.1 Installing Python

If Python is not installed on your system, it can be downloaded from

```
http://www.python.org/
```

Follow the installation instructions for Python given on the Python web page. For Debian (Ubuntu) users, the package to install is named `python`.

### E.2 Installing UFC

UFC follows the standard installation procedure for Python packages. Enter the source directory of UFC and issue the following command:

```
# sudo python setup.py install
```

This will install the UFC utility Python package in a subdirectory called `ufc` in the default location for user-installed Python packages (usually something like `/usr/lib/python2.5/site-packages`). In addition, the header file `ufc.h` will be installed.

To install UFC under a different directory, for example if you don't have root access to the system you're using, type

```
# python setup.py --prefix=~/.local install
```

This installs the UFC header file in the directory `~/.local/include` and the Python utilities under `~/.local/lib/python`. If you use this option, make sure to add `~/.local/lib/python` to the environment variable `PYTHONPATH` and to add `~/.local/lib/pkgconfig` to the `PKG_CONFIG_PATH` environment variable.

# Appendix F

## License

The UFC specification, and in particular the header file `ufc.h`, is released into the public domain.





# Index

- hexahedron, 24
- interval, 24
- quadrilateral, 24
- tabulate\_tensor, 39, 40
- tetrahedron, 24
- triangle, 24
- ufc::cell\_integral, 39
- ufc::cell, 26
- ufc::dof\_map, 33
- ufc::exterior\_facet\_integral, 39
- ufc::finite\_element, 29
- ufc::form, 42
- ufc::function, 28
- ufc::interior\_facet\_integral, 40
- ufc::mesh, 25
  
- assembly, 18
  
- Cell shapes, 24
- cell tensor, 20
- code generation, 107
  
- dependencies, 109
- design, 23
- discretization, 16
  
- example code, 77
- exterior facet tensor, 20
  
- FFC, 9, 78
- finite element, 14
- form compilers, 9
  
- global tensor, 17
  
- hexahedron, 52
  
- installation, 109
- interface, 23
- interior facet tensor, 20
- interval, 48
  
- license, 111
  
- mesh entity, 53
  
- numbering, 53
  
- Poisson's equation, 13, 77
- Python utilities, 107
  
- quadrilateral, 50
  
- reference cells, 47
  
- SyFi, 9, 91
  
- test space, 14
- tetrahedron, 51
- topological dimension, 53
- trial space, 14
- triangle, 49
  
- ufc.h, 24
  
- variational form, 15
- variational problem, 13
- vertex numbering, 54